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No. 31.

WAVE ACTION

IN RELATION TO

ENGINEERING STRUCTURES.

BY

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WAR DEPARTMENT,
OFFICE OF THE CHIEF OF ENGINEERS,
Washington, January 18, 1904.

SIR: 1. I have the honor to submit herewith a paper on "Wave Action in Relation to Engineering Structures," with 38 accompanying plates, figures, and photographs, by Capt. D. D. Gaillard, Corps of Engineers, U. S. Army.

2. This paper contains information of great value to officers of the Corps of Engineers and to the engineering profession at large, and I have the honor to recommend that authority be granted by the Secretary of War to have it printed, with its accompanying plates, figures, and photographs, at the Government Printing Office, as a Professional Paper of the Corps of Engineers, and that 1,000 copies be obtained for the use of the Engineer Department upon the usual requisition. I certify that these illustrations are necessary to a full understanding of the paper and relate entirely to the transaction of public business.

3. In a letter which is on file in this office, Captain Gaillard has tendered to the Engineer Department the use of this paper, with free consent to its publication, expressing the desire, however, to reserve to himself all other rights therein, which he proposes to protect subsequently by copyright, if allowable under the laws relating to copyrights.

Very respectfully, your obedient servant,

G. L. GILLESPIE,
Brig. Gen., Chief of Engineers,
U. S. Army.

Hon. ELIHU ROOT,
Secretary of War.

[First indorsement.]

WAR DEPARTMENT,
January 21, 1904.

Approved as within recommended.

ROBERT SHAW OLIVER,
Assistant Secretary of War.

PREFACE.

While engaged upon works of river and harbor improvement on the South Atlantic coast and on Lake Superior, the writer at times observed such uniformity of wave action under similar conditions that he could not escape from the conviction that the generally accepted idea of the hopelessness of attempting to compute wave force was probably largely due to the fact that so few observations upon shallow-water waves had ever been attempted.

As a rule, an engineering structure subject to wave action is exposed only to the attack of shallow-water waves; yet, strange as it may seem, the number of recorded measurements of waves of this class is insignificant when compared with that of deep-water waves.

An investigation of the subject of wave action, which, for lack of time and of opportunity to visit reference libraries, has not been as exhaustive as was desired, has shown that while the theory of wave action has been ably treated by a number of eminent mathematicians, yet their individual discussions pertain as a rule to certain particular phases of wave action, and are embraced in so many different volumes, that the work of comparing theoretical and observed wave characteristics is rendered unnecessarily great.

In the mathematical treatment of the subject which follows, the writer lays no claim to originality except in the cases of equations (17), (16A), and (9*b*), and in the application of equation (24) to the measurement of the force exerted by breaking waves.

All plates are original, except Pl. IV, as are also most of the photographs and tables and many of the figures.

No attempt has been made to follow out step by step the mathematical deductions relating to the subject of wave action; but, instead, only the results deduced are given, and they have been arranged in such order as to facilitate comparison between theoretical and observed results.

Had the writer remained on duty in the Lake Superior district during the season of 1903, it had been his intention to continue observations with diaphragm dynamometers, placed one above another from the bottom up, and connected with gauges so constructed as to reduce the momentum of moving parts to a minimum.

It had been further intended to provide one or more of the gauges with clock-work mechanism, by means of which a continuous record of pressures could have been obtained.

The diaphragm dynamometer, recording as it does the impact of individual waves, and being adapted for use at any height from the bottom up, gives promise of being a valuable adjunct in the investigation both of wave impact and of static pressures due to waves.

It is therefore a source of regret to the writer that so little time was available, after these instruments were installed, for investigating their action during storms, and adopting such modifications or improvements in construction as might be shown to be desirable.

It had been his intention not to submit the results of his observations upon shallow-water waves until he had secured much fuller observations with diaphragm dynamometers, but his assignment in the spring of 1903 to duties of a purely military character precluded the possibility of securing such observations, and it was therefore thought best to present what had thus far been obtained, hoping that others would continue investigations in this important field, which has been as yet so scantily covered.

In conclusion, the writer desires to express his hearty appreciation of the valuable assistance rendered him in the preparation of what follows.

Especially is he indebted to Messrs. J. H. Darling and Clarence Coleman, United States assistant engineers, without whose most efficient assistance it would have been almost impossible to secure many of the observations embraced in the various tables.

He is also greatly indebted to the hydrographer in charge, Hydrographic Office, United States Navy; Commander Z. L. Tanner, U. S. Navy, retired; Prof. A. G. Greenhill, R. M. A., Woolwich, England; Mr. William Shield, London, England; Mr. Theodore Cooper, New York; Majors Thos. W. Symons,

James G. Warren, and William C. Langfitt, Corps of Engineers, U. S. Army, and Mr. James Page, United States Hydrographic Office, for data furnished; to Naval Constructor D. W. Taylor, U. S. Navy, for data furnished and for valuable suggestions; to Majors John G. D. Knight and Lansing H. Beach, and to Capts. Charles H. McKinstry and Edgar Jadwin, Corps of Engineers, U. S. Army, and Mr. D. E. Hughes, United States assistant engineer, for assistance in reviewing manuscript, and to Mr. Frank Henrich, nautical expert, United States Navy, for German translation.

D. D. GAILLARD,
Captain, Corps of Engineers, U. S. Army.

VANCOUVER BARRACKS, WASH.,
September, 1903.

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WAVE ACTION IN RELATION TO ENGINEERING STRUCTURES.

CHAPTER I.

IMPORTANCE OF SUBJECT. DEFINITIONS. GENERAL EXPLANATION OF WAVE MOTION. HOW CAUSED. CLASSIFICATION OF WAVES.

IMPORTANCE OF SUBJECT.

A knowledge of wave action is of great importance to engineers engaged in planning marine structures, such as breakwaters, piers, jetties, groins, docks, wharves, light-houses, works for shore or bank protection, revetments for dams, reservoirs, etc., and although in the present state of knowledge it is not possible, from previous observations, to compute, mathematically, the exact wave forces which will act upon every part of the proposed structure, yet what has already been determined by careful investigators, if properly applied, will prove of material assistance in the planning and execution of the work. The increase in the knowledge of wave action, during the past hundred years, has been so great that it is, perhaps, not too much to hope that under certain conditions the action of wave forces upon engineering structures may yet be determined in advance with as much accuracy as are other forces to which such structures are ordinarily subjected.

Before proceeding further it is advisable to define some of the terms commonly employed in discussing the subject of wave action.

DEFINITIONS.

Breaker, comber.—A wave which breaks in shallow water or upon reaching shore.

Cycloid, common.—The curve described by a point on the circumference of a circle rolling on a straight line.

Cycloid, prolate.—That described by a point within the circle and in its plane.

Depth, general depth.—The distance from the surface (supposing all wave action to have ceased) to the bottom.

Fetch.—The distance across open water from the windward shore.

Line of orbit centers.—The horizontal line connecting the centers of the orbits of those particles in the wave the dimensions of whose orbits are the same.

Orbital velocity.—The velocity of a particle in its orbit.

Ripple.—The smallest class of waves and one in which the force of restoration of the particles is chiefly the surface tension of the water.

Seiche (pronounced sāsh).—A wave or fluctuation of very great length, found in large fresh-water lakes or land-locked seas, and attributed to sudden local variations in barometric pressure.

Still-water level.—(See *Undisturbed water level.*)

Surf.—A collective name for breakers.

Swell, ground swell, roller.—A long, unbroken wave, which rolls in after a storm, or as the result of a distant storm.

Trochoid.—As ordinarily used in wave motion, a curve identical with the prolate cycloid.

Undisturbed water level.—The level at which the water would stand if wave action instantly ceased.

Wave.—A disturbance of the surface of a body in the form of a ridge and trough, propagated by forces tending to restore the surface to its figure of equilibrium, the particles not advancing with the wave.

Wave crest.—The top or highest part of a wave.

Wave, deep-water.—One which is being propagated in water of a depth greater than half of the wave length.

Wave, forced.—One upon which the force causing it continues to act, as in the case of a wave during a storm.

Wave, free.—One upon which the force causing it no longer acts, as when swells roll in after the storm ceases.

Wave height.—The vertical distance from the highest to the lowest point of the wave.

Wave hollow or trough.—The lowest part of a wave.

Wave length.—The distance between the crests or hollows of two adjacent waves, or, more generally, the distance be-

tween any particle of the disturbed medium and the next which is in the same phase.

Wave, negative.—One which lies wholly below the general water level.

Wave, oscillatory.—One in which each particle describes a closed orbit around its position of rest, without advancing in the direction of wave travel; usually applied to waves in deep water.

Wave period.—The time between the passage of successive wave crests or hollows.

Wave, positive.—An advancing elevation unaccompanied by a depression.

Wave, shallow-water.—One which is being propagated in water of a depth less than half of the wave length.

Wave, standing.—One brought to a standstill by water flowing in the opposite direction.

Wave, tidal.—The wave of the tide; also applied to waves of unusual size and of rare occurrence, as those produced by earthquakes. A species of tidal wave is the "Bore"—a high-crested, roaring wave caused by the rushing of flood tide up a river or estuary, or by the meeting of tides.

Wave of translation.—One which, after its passage, leaves the particles shifted in the direction of wave travel.

Wave velocity.—The rate at which a wave advances; generally expressed in feet per second.

White-cap.—A foam-crested wave caused by wind, of velocity sufficient to blow the wave crest forward and over.

GENERAL EXPLANATION OF WAVE MOTION.

As water possesses the characteristic of mobility, any disturbance communicated to one of its particles is transmitted to contiguous particles, and through them to others more remote, causing oscillatory movements known as waves.

At first sight the entire mass of water composing a wave appears to be moving in the direction of wave travel, but upon closer investigation it will be found that such is not the case. If a body floating upon the surface of the water be observed carefully, it will be seen to rise, move forward, and sink when on the upper portion of the wave, and to continue to sink, move backward, and rise again when on the lower portion of the wave, but without appreciable movement

in the direction of wave travel, except such as may be due to the action of wind or of currents. Each particle moves about its position of rest in a closed orbit, in a manner consistent with the movement of all other particles in the wave. How this is accomplished is shown in figs. 1 and 2, p. 16, which are modifications of Webers' diagram of an oscillatory wave; the particles moving in circular orbits in the same direction as the hands of a clock, and the wave advancing in the direction

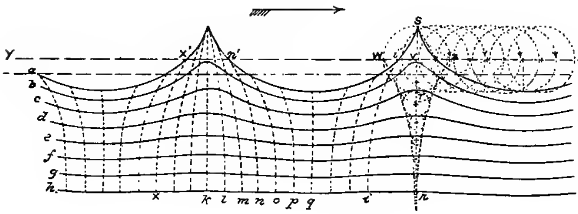


Fig. 1.

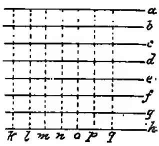


Fig. 2.

shown by the arrow. *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, etc., fig. 2, represent horizontal, and *k*, *l*, *m*, *n*, *o*, *p*, *q*, etc., vertical filaments of water in a state of rest. The positions of the corresponding filaments during the passage of a wave are shown in fig. 1. In this figure the filament *a* is represented by the common cycloid, and all other horizontal filaments by prolate cycloids. The dimensions of the orbits of the particles decrease rapidly below the surface, as indicated by the limiting lines *rz* and *rw* in the figure.

Those particles which lie in the same vertical filament when at rest, arrive at the lowest point of their orbits at the same instant when wave motion is in progress, taking the position shown at *q*. When the wave advances, the filament takes successively the positions *p*, *o*, *n*, etc., the upper portion bending over toward the wave crest until at *k*, directly under the crest, it becomes vertical. After the crest has passed, the filament again inclines toward it until the next succeeding trough arrives, when it again becomes vertical.

When the wave occupies the position shown in fig. 1, all particles between the filaments *xx'* and *nn'* have motion in the direction of wave travel, and those between *nn'* and *ii'* in the contrary direction.

When a wave travels from deep water into water of gradually decreasing depth, a change in form takes place. Owing

to friction upon the bottom, the velocity of the wave and the distance from hollow to hollow decreases, while the wave height for a time increases. The front of the wave becomes steeper and steeper, and the velocity of its depressed portion decreases to such an extent that the greater velocity of the particles at the top carries them forward, orbital motion ceases, and the crest falls over and is carried forward, breaking into a foaming mass of water rushing shoreward.

The direction of the wave crest in deep water is at right angles to that of the wind, but on approaching shore it tends to become parallel to the shore line, due to the fact that if the crest approaches obliquely the end closest inshore reaches shallow water first, and has its velocity retarded by friction on the bottom, causing the more distant part of the crest to gain upon it and to swing around parallel to the shore.

Most waves are caused by the action of the wind, and the manner in which this is accomplished appears to be as follows: before the wind begins blowing, the water may be perfectly still, but as soon as the wind commences, a change quickly takes place.

Wind velocity is never constant, but acts in puffs and gusts, causing varying pressures to be exerted upon the surface of the water, which, under this action, soon becomes ruffled, presenting undulations upon which the wind can act directly. The elevated portions of the undulations thus formed are fully exposed to the wind, while the depressions are partially or entirely screened by the crests in rear. These undulations or waves are continually changing in form and increasing in size under this action until limited by the "fetch" and the velocity of the wind.

When the velocity of the wind is considerably greater than that of the waves, it acts both by friction and by direct pressure upon the crest of the latter, exerting a force tending to push forward the top too strongly for continuous undulation. The crest is then carried forward too far, and the wave breaks, as is seen in the case of "white caps" and of large waves which break during storms in water of great depth.

On large bodies of water storm waves sometimes reach shore in advance of the storm itself, giving warning of the approach of the latter.

This, as will be explained later, is probably due to the rotary motion of the wind around the storm center.

Waves may be produced by other causes than wind action, as, for example, by vessels under way, submarine explosions, barometric fluctuations, the attraction of the sun and moon, earthquakes, etc.

For purposes of investigation, waves in troughs or narrow channels have in some cases been formed by suddenly admitting a body of water of higher level or by rapidly withdrawing a part of the water, and in other cases by the introduction or removal of a solid body from the water.

It must not be supposed that the waves commonly seen during storms have much regularity of form. On such occasions it is almost impossible to follow a single wave with the eye for any length of time before it seems to disappear and become merged into another. It will be noticed that waves of many different sizes exist, and that sometimes one or more systems of small waves are seen superimposed upon the larger ones and traveling in the same direction. In other cases, two simultaneously existing systems of waves may be seen traveling in different directions. In yet other cases, waves reflected from the shore or from artificial constructions meet the incoming waves, the interference thus produced sometimes augmenting and sometimes neutralizing the opposing waves, often producing such a chaotic movement of the water that all regularity of oscillation is destroyed. It is a matter of common belief that at a certain regularly recurring interval there comes a wave of unusual size. In most cases this is only partially true, for while unusually large waves appear at intervals, yet the regularity of these intervals is by no means as great as is commonly supposed. The unusual size of these waves is due to the conjunction of two or more waves of different systems.

Although theoretically when waves exist in deep water there is no actual motion of translation of the particles, yet it is now very generally recognized that all storm waves are to a certain extent waves of translation, however great the depth of water. Waves of all classes become waves of translation upon reaching shallow water.

The waves which approach most nearly the theoretical form are the long, regular swells or rollers, which are seen after a

storm has partially or wholly subsided, or as the result of a distant storm. Waves of the same class are also occasionally seen as the advance guard of an approaching storm.

CLASSIFICATION OF WAVES.

Waves have been variously classified by different writers as follows:

(*a*) With respect to the continuance or noncontinuance of the generating force, as *free* or *forced*.

(*b*) With respect to periodicity, as *solitary* or *successive*.

(*c*) As regards the position of the wave with respect to the water surface, as *ordinary*, *positive*, and *negative*.

(*d*) In respect to the orbital motion of the particles, as *oscillatory waves* or *waves of translation*.

(*e*) In regard to size, as *tidal waves*, *seiches*, *storm waves*, and *ripples*.

(*f*) As to appearance, as *ordinary waves*, *white caps*, *swells*, and *breakers*.

These classifications, however, are largely arbitrary, and do not serve to define the wave completely; for example, a wave may at the same time be free, solitary, positive, and a wave of translation.

Storm waves, swells, and breakers comprise the principal classes of waves which act against engineering structures.

Tidal waves, seiches, and ripples develop no injurious wave action proper against these structures, and any damage caused by the two former will be due either to the change in water level or to the currents resulting.

The action of positive and negative waves principally concerns naval architecture and canal navigation.

CHAPTER II.

Investigations of Waves.

By Franz Gerstner, Ernst and Wilhelm Weber, J. Scott Russell, George B. Airy, Thomas Stevenson, Henri Bazin, P. G. Tait, Rankine, Stokes, Earnshaw, Kelland, Green, Greenhill, Gatewood, and Cooper. Observations taken from vessels at sea by Lieutenant Paris, officers of the U. S. Navy, Abercromby, Doctor Schott, Lieutenant Gassenmayr, and Doctor Cornish. Observations in the United States—at Oswego, Milwaukee, St. Augustine, and on Lake Superior.

The character of the orbits of the particles and the form of the wave have been investigated both mathematically and experimentally by many scientists of high repute. Unfortunately, however, actual wave motion in an open sea presents conditions of such complexity as to prevent, in the present state of knowledge, any general mathematical deductions which will prove applicable to every case which may arise.

Consequently, the results obtained have generally been deduced under certain conditions or suppositions which often materially limit their application and prevent their use in the particular case which confronts the engineer.

He is then forced either to rely upon the results of experiments or to plan his structure in accordance with the best practice of his profession in cases believed to be similar to the one in hand.

The latter would be the safer practice did true similarity exist; but in scarcely any branch of engineering are the forces developed and the methods and directions of their application more variable than in the case of wave action, and often a structure which is stable as regards the action of wave forces in one locality would be destroyed if erected in another, although on superficial investigation the two sites might appear to be equally exposed.

Careful investigations of the character of wave action, the intensity of wave forces, and the methods, directions, and extent of their application are of the greatest value to the

engineering profession, and it is to be regretted that the number of experimental investigators of the subject has been so limited in the past, especially in our own country.

The field for investigation is wide and much of it has been but scantily covered up to the present time.

INVESTIGATIONS OF WAVE ACTION.

Franz Gerstner.—Among the more important of the earlier theoretical discussions of the subject of wave motion are those by Lagrange, La Place, Poisson, and Cauchy. To Franz Gerstner, however, is due the honor of having first solved the problem of wave motion by assuming an actual displacement of particles of water. His discussion of the subject was published in the Transactions of the Royal Bohemian Scientific Society for 1802. Among other things he states that on the surface the wave is a prolate cycloid, with the common cycloid as the limit at breaking, and that the orbit of a particle is a circle.

Ernst and Wilhelm Weber.—In 1825 the two brothers, Ernst Heinrich and Wilhelm Weber, published a work called "Wellenlehre auf Experimente gegrundet" (Theory of Waves Founded upon Experiment), containing an abstract of all preceding theories and experiments of note known to the writers, and describing experimental investigations made by themselves.

This work formed a most important contribution to the knowledge of wave motion.

In making their experiments they used in one case a narrow trough with glass sides, 5 feet 4 inches long, 8 inches deep, and about half an inch wide. In another case the trough was 6 feet long, 2½ feet deep, and a little over an inch wide. In the former case the glass sides were continuous, and in the latter case the sides were of wood with six openings covered by glass.

Experiments were made upon waves in brandy, in water, and in mercury. Most of the observations, however, were made upon waves in water, generated by plunging a glass tube into the fluid, raising the latter in the tube by suction, then allowing it to drop. In some cases the wave was observed after being reflected from one end of the trough, but in most cases soon after it was formed and before reflection took place.

The form of the wave was obtained by plunging a slate sprinkled with flour into the trough in a position parallel to the sides of the latter, and rapidly withdrawing it, thus obtaining a satisfactory trace of the front of the wave and a less satisfactory trace of the back. These observations showed that the vertical section of a wave at the surface was a cycloidal curve, and that when the height of the wave was large, as compared with the depth of water, its front was much steeper than its back.

The orbital motion of the particles was noted by placing in the water a number of small particles of about the same specific gravity as the water itself, and observing their movements through the glass sides either with the naked eye or with an instrument. By these methods they arrived at the following conclusions:

Where a wave ridge was followed by an equal wave hollow every particle moved in an ellipse, or a curve as closely approximating an ellipse as could be judged by eye, the major axis being horizontal, and the motion of the particles on the highest part of the ellipse being in the same direction as that of the wave, and at the lowest part in the opposite direction.

When a large wave ridge is followed by a small wave hollow the motion of a particle is represented as follows:  and when a small ridge is followed by a large hollow it is like this: .

The horizontal motion of the particles was found to diminish somewhat for the deeper particles.

The vertical motion diminished with the depth to such an extent that on approaching the bottom the ellipse became almost a horizontal line.

When two equal waves met it was found that the motion of each particle was backward and forward in a straight line.

The velocity of the wave was determined to be independent of the specific gravity of the fluid, but increased with an increase of the depth of the fluid in the trough, and was greater for a large wave than for a small one. The law of variation of the velocity in these cases was not determined. Time was taken with a watch by means of which an interval as small as one-sixtieth of a second could be determined.

The deductions which precede are free from theory, and are based solely upon experimental data.

Although the great value of the experiments just described is generally recognized, yet certain of the details have not escaped criticism, as, for example, the narrowness of the troughs used tending to magnify the effect of friction, or irregularity of the sides, the short distance between the ends of the trough necessitating rapid observations soon after the formation of the waves.

J. Scott Russell.—Mr. J. Scott Russell, a noted British engineer, made valuable investigations upon the subject of waves at the instance of the British Association for the Advancement of Science, which are published in the reports of the Association for 1837 and 1844. These investigations were originally undertaken in the interest of canal navigation. Special apparatus was constructed for the proposed experiments, which answered admirably the purpose for which designed. Observations were made in a rectangular trough 20 feet 7.3 inches in length, 1 foot in width, and a little over 7 inches in depth. Only 20 feet of the trough was traversed by waves, the remaining part being used for generating them. This was accomplished by one of the following methods:

(*a*) A gate or sluice was placed 7.3 inches from one end of the trough, forming a small reservoir, which was filled to a known height, greater than that of the water level in the trough proper. When the gate was raised, the water from the reservoir rushed into the trough, causing a swell, which was propagated along the trough as a wave. The gate was then quickly depressed, forming a vertical face similar to that at the opposite end of the trough.

(*b*) A vertical rectangular box, without top or bottom, which occupied the whole or a part of the reservoir end of the trough, was filled with water to a definite height and then lifted up, permitting the water to rush out below its lower edge, causing a wave, as in the previous case.

(*c*) The gate previously described was agitated by the hand, forming a wave.

(*d*) In some cases the waves were generated by pressing a solid body into the water; (*e*) and in other cases by withdrawing it from the water.

The method used for obtaining wave velocity was novel and ingenious.

Mr. Russell realized that the length of the trough was not

sufficient to permit of great accuracy in determining the velocity of a wave, but noticed that upon reaching the vertical ends of the trough the wave was reflected without alteration of form, and could be observed as well as if the trough itself had been correspondingly prolonged. Making use of this fact, some waves were observed after 60 reflections, or after traveling a total distance of 1,200 feet. Observations could be taken at three points along the trough; and in the cases just described a single wave was observed as often as 180 times during its 1,200 feet of travel.

Equal ingenuity and ability were shown in devising the method of determining the precise instant at which the crest of the wave passed a given point. "The flame of a candle, placed above the trough and at a small horizontal distance from it, was reflected by a mirror in an inclined position, downward to the water, then by the surface of the water it was reflected upward, and being received upon another inclined mirror was reflected to the eye of an observer, who viewed it through an eye tube, furnished with an internal wire and a more distant mark for directing the observer's eye. When the water was at rest, or when the horizontal surface at the top of the wave was passing under the mirror, the candle was seen in the center of the eyepiece. When an inclined part of the wave, either the anterior or the posterior, was passing, the candle was seen on one or the other side of the eye tube."

By this method it was possible to observe with accuracy the passage of the highest part of a wave, the length of which was 3 feet, and the wave height but one-tenth of an inch.

The wave length was determined by adjusting two conical points almost in contact with the water surface, so that the most advanced part of the front of the wave touched one point at the same instant that the extreme rear of the wave left the other.

The wave height was determined by means of small pipes let into the side of the trough, and passing upward on the outside.

Mr. Russell's experiments were all made upon single waves of the class denominated by him "The great primary wave," and as a result of his investigations he concluded:

(*a*) That the wave which he called "a great primary wave

of translation" differed in its phenomena and laws from the undulatory and oscillatory waves, which alone had been investigated previously.

(*b*) That the velocity of such a wave in a channel of uniform depth is equal to that acquired by a heavy body falling through a height equal to half the depth, reckoned from the top of the wave to the bottom of the channel.

(*c*) The velocity of the primary wave is unaffected by the method in which the wave is formed, and neither its form nor velocity is affected by the form of the body generating it.

(*d*) This wave differs from all others in the character of the motion of its particles. As the wave progresses the particles are elevated, moved forward in the direction of travel of the wave, and deposited at rest in advance of their original positions. No oscillatory or backward movement occurs, and the extent of the forward movement is equal throughout the whole depth. The motion of translation commences when the anterior surface is vertically over a particle, and increases in velocity until the wave crest passes over the particle, and decreases to zero when the posterior surface has passed.

(*e*) The form of the wave is that of the cycloid, prolate when the wave height is small in proportion to its length, and approaching the common cycloid as the wave height increases, until upon reaching this form the wave breaks.

(*f*) In a channel the cross section of which is trapezoidal or triangular the wave velocity is that due to a heavy body falling through a height equal to one-third the maximum depth.

(*g*) The height of the wave may be increased by propagation in a channel of uniform depth which is continually becoming narrower; the increase in height varying nearly in the inverse ratio of the square root of the breadth of the channel.

(*h*) When the wave is propagated in a channel of uniformly diminishing depth, it will break when its height above the surface of the level fluid is equal to the depth.

Mr. Russell also investigated what he calls the "negative wave of translation"—i. e., one which is propagated as a depression in the surface of the water. His conclusions in regard to this wave were—

(*a*) That the negative wave is a wave of translation, the

movement of the particles being in the contrary direction to that of wave travel.

(*b*) Its anterior form and path of translation are nearly those of the positive wave reversed.

(*c*) Its velocity is less in considerable depths than that due to half the depth, measured from the lowest point, and is also less than that of a positive wave having the same total height.

(*d*) The negative wave can not be generated alone, but is always followed by secondary waves.

It having been suggested that positive and negative waves might be corresponding parts of an ordinary wave, Mr. Russell gives certain conclusions, founded upon his experiments, showing that this is not the case.

(*a*) When an attempt is made to propagate them, so that one shall be complementary to the other, they immediately separate.

(*b*) If the positive wave is in front, it travels with greater velocity, rapidly moving away from the other.

(*c*) If a positive wave be generated behind a negative wave, it will overtake it.

(*d*) If a positive and negative wave of equal volume, and traveling in opposite directions, meet, or if a positive wave overtakes a negative wave of equal volume, they neutralize one another and both cease to exist. But if either be larger, the neutralization is not complete, and the remainder is propagated as a wave of the larger class.

As one result of his investigations, Mr. Russell ascertained that the most economical speed for a canal boat is the same as that of the wave of translation to which it gives rise—i. e., the velocity due to one-half the depth. If a boat moves at this speed, it remains on the crest of the wave; but if it moves at greater or less speed, it generates new waves at the expense of power.

Although interesting, this conclusion is of little practical value, for in a canal of but 5 feet in depth it would require a velocity of over 9 miles an hour—a speed too great for animals, and one which would tend to destroy the banks of an ordinary canal.

In addition to those already described, Mr. Russell made observations upon waves in navigable canals and in the sea.

In the former case the results showed general agreement with conclusions already stated, but in the latter case the velocity observations were unsatisfactory. As the wave lengths were not determined, this is not surprising.

Mr. Russell's experiments, although showing great ability, ingenuity, and painstaking care, lose much of their value to engineers from the fact that they do not deal with the class of waves usually encountered—i. e., those formed by the action of the wind.

George B. Airy.—Prof. George Biddle Airy, former astronomer royal, contributed to the Encyclopædia Metropolitana, volume V of Mixed Sciences, under the title of "Tides and waves," a very valuable and complete mathematical discussion of the subject.

Most of his results are deduced for "canals of uniform depth and uniform breadth, whether the waves be short or long, the motions of the particles being supposed small."

He shows that in this case:

(*a*) The motion of every particle is in a circle, or an ellipse whose major axis is horizontal; if in a circle, its motion in that circle is uniform; if in an ellipse, the horizontal motion is, at any instant of time, in the same proportion to the whole horizontal motion as in a circle, and the vertical motion is also in the same proportion to the whole vertical motion as in a circle; the greatest horizontal motion forward occurring when a particle is at the top of the wave; the greatest horizontal motion backward occurring when the particle is at the bottom of the wave, and the horizontal motion being zero when the particle is at its mean elevation.

(*b*) When the depth is great in comparison with the length of the wave, as in the case of ordinary waves in the open sea, the motion of the water at any great depth below the surface is wholly insignificant in comparison with that at the surface.

As the depth below the surface increases in arithmetical progression, the motion diminishes in geometrical progression, and at a depth equal to the length of the wave the motion is diminished to $\frac{1}{535.4}$ of that at the surface.

(*c*) On the same supposition the greatest horizontal motion of any particle is equal to its greatest vertical motion, except for those particles very near to the bottom, where the whole motion is insensible.

(*d*) When the length of the wave is great in comparison with the depth of the water, as in the case of tide waves, the horizontal motion is sensibly the same from the surface to the bottom, and the vertical motion for different particles varies in the same proportion as their height above the bottom.

(*e*) On the same supposition, the vertical motion of the superficial particles is very much less than their horizontal motion.

(*f*) When the length of the wave is not greater than the depth of the water, the velocity of the wave depends (sensibly) only on its length, and is proportional to the square root of its length.

(*g*) When the length of the wave is not less than a thousand times the depth of the water, the velocity of the wave depends (sensibly) only on the depth, and is proportional to the square root of the depth, being the same as the velocity of a free body falling from rest under the action of gravity through a height equal to half the depth of the water.

(*h*) For intermediate proportions of the length of the wave and the depth of the water, the velocity of the wave is expressed by a more complicated formula, which will be referred to hereafter.

(*i*) When two waves equal in period and equal in magnitude meet each other each particle moves in a straight line.

(*j*) In a canal of uniform breadth, but of variable depth, and with gravity alone acting, it is impossible that there can be a series of waves, consisting of oscillatory motion of the particles, which satisfy the conditions of continuity and equal pressure, but in such a case the condition of continuity will cease, i. e., the water will become broken. He regards this as the explanation of the broken water seen upon the edge of a shoal or ledge of rocks which may be deeply covered with water.

When waves in an open sea are reflected from a vertical plane surface, it is shown that they give rise to another system of waves having the same degree of inclination to the reflecting surface, but inclined in the opposite direction, the undulation at contact with the reflecting surface being twice the elevation of the unreflected wave.

Professor Airy also investigated, mathematically, the case of a single discontinuous wave traveling along a canal, and

found that if the single wave is moderately long a small force will maintain it as a discontinuous wave, but if it be short the force must be considerable.

He also found that in a channel of uniform breadth, whose depth diminishes gradually, the height of the wave varies inversely as the fourth root of the depth.

These investigations are of great interest, because the conclusions deduced mathematically agree as well as could be reasonably expected with those reached experimentally by previous investigators.

Professor Airy was of the opinion that the agreement of his formulas with the results of Mr. Scott Russell's experiments was as close as could have been expected, but Mr. Russell expressed himself most emphatically as being of a different opinion.

Thomas Stevenson.—From an engineering point of view, probably the most valuable contribution to the knowledge of wave action is due to the investigations of the eminent British engineer, Thomas Stevenson, the results of which are described in his valuable work, The Construction of Harbours, first published in 1864, and since revised.

Mr. Stevenson's conclusions are based upon numerous observations, some of which were made upon waves in canals, some in fresh-water lakes, but by far the greater number upon sea waves. His formulæ and conclusions are founded solely upon experimental data, and are particularly valuable on account of the long period over which the investigations extended and the large number of observations upon which they are based.

He deduced empirical formulæ for the height of waves due to the "fetch;" for the reduction of waves by lateral deflection, and for their reduction in height after passing into close harbors.

Among other subjects his investigations included the maximum height of waves in large bodies of water; their length and velocity; the relation between the wave height and the depth in which the wave breaks; the force of waves; its character; the manner in which it acts; and its observed effects.

Mr. Stevenson appears to have been the first to attempt systematic measurements of wave force. For this purpose he

devised the type of marine dynamometer shown on page 145 by means of which numerous observations covering long periods were secured.

Although Mr. Stevenson began his dynamometer observations in 1842, and although the subject is of great importance to engineers, it is rather remarkable up to the present time how few persons have directed their attention to this branch of investigation.

Mr. Stevenson's treatment of the subject of wave action is too extended to be described properly at this point, and will be taken up more in detail hereafter.

Henri Bazin.—In 1859 Henri Bazin, a French officer of engineers, instituted a series of experiments upon waves in an open channel 6½ feet wide, and having a bottom inclined about 1½ feet in a thousand, the channel being straight and of uniform width.

The results of his experiments were analyzed and discussed in a memoir presented to the French Academy, and published in 1865.

He found that in the case of the isolated wave (positive wave) it moved as a symmetrical undulation, the height increasing, and the velocity decreasing as the depth diminished, till finally it broke before the depth had decreased to the height of the wave. The velocity observed agreed well with the law enunciated by Scott Russell.

Bazin also made experiments upon the negative wave formed by withdrawing a certain amount of water from the channel. His observations upon the velocity of this wave did not agree with the results obtained by Scott Russell, the velocity found being nearly that due to a heavy body falling through a distance equal to half the depth of water, reckoned from the lowest point of the wave hollow. Bazin ascertained that negative waves were propagated with the same velocity in moving as in still water, proper allowance being made in the former case for current velocity. This wave, however, diminished and died out more rapidly than the positive wave in still water. In moving water both kinds of waves diminished more rapidly than in still water.

Von G. Hagen.—Investigations upon the subject of waves were made by Von G. Hagen, and are embraced in his Trea-

tise on Waves in Water of Uniform Depth, Berlin, 1862, and Seeufer-und Hafenbau, Von G. Hagen, Berlin, 1863. He showed that the line which unites points which are in the same phase of revolution is a prolate cycloid, which becomes more prolate as the depth increases, and at the surface approaches the common cycloid, the orbit of a particle being a circle.

Hagen verified Franz Gerstner's theory of oscillatory waves, and advanced a theory of his own for waves in shoal water. His conclusions in respect to the latter class of waves, however, do not agree satisfactorily with those of most previous investigators.

W. Froude.—Mr. W. Froude, in an article published in the Transactions of the Society of Naval Architects, 1862, gives the form of an oscillatory wave in deep water, as the prolate cycloid, with the common cycloid as its limit just before breaking. He gives the orbit of a particle as a circle, and obtains a mathematical expression for the energy due to motion of all particles between the surface and any given depth to which orbital motion extends. (See p. 40.)

P. G. Tait.—Prof. P. G. Tait, in the Encyclopædia Britannica, Volume XXIV, gives a very valuable theoretical and mathematical discussion of the subject of waves.

Professors Rankine and Stokes.—Prof. William John Macquorn Rankine and Prof. Gabriel G. Stokes have in various articles discussed mathematically the subject of wave motion very clearly and comprehensively.

Other theoretical discussions of the subject.—Long waves have been very ably discussed mathematically by Earnshaw, Kelland, and Green. Prof. A. G. Greenhill has furnished a very interesting mathematical discussion of waves in Volume IX of the American Journal of Mathematics.

Naval Constructor R. Gatewood, U. S. Navy.—Naval Constructor Gatewood, U. S. Navy, has prepared a very clear and complete mathematical discussion of the subject of "Water waves," embracing deep-water waves, shallow-water waves, compound waves, waves in groups, etc.

Theodore Cooper.—Mr. Theodore Cooper (Mem. Am. Soc. C. E.), in 1896, contributed to the transactions of the society an interesting article entitled "Some general notes on ocean waves and wave force."

OBSERVATIONS TAKEN FROM VESSELS AT SEA.

Lieutenant Paris.—During the cruises of the *Dupleix* and *Minerve*, 1867–1870, Lieutenant Paris, of the French navy, made valuable observations on wind and sea.

Officers of the United States Navy.—Between 1883–1887, officers of the United States Navy obtained a large and interesting series of observations upon deep-sea waves in most of the principal oceans of the globe.

Hon. Ralph Abercromby.—In 1885 Hon. Ralph Abercromby secured a number of valuable observations upon deep-water waves. These observations were taken on board the S. S. *Tongariro* in various parts of the South Pacific between New Zealand and Cape Horn.

Dr. Gerhard Schott.—In 1891–92 Doctor Schott made an interesting series of measurements of dimensions of deep-water waves while on board the sailing ships *Robert Rickmers* and *Peter Rickmers*, of Bremen, during a voyage in the interest of hydrographic and maritime meteorological investigations. The Cape of Good Hope was doubled twice, for which reason most of his observations were made in south latitude.

Lieut. Oskar Gassenmayr.—In 1895 Lieut. Oskar Gassenmayr, H. I. M. S. *Donaŭ*, obtained a valuable series of measurements of deep-water waves, most of the observations being taken in the South Atlantic.

Dr. Vaughan Cornish.—Dr. Vaughan Cornish, of London, in 1900 made observations upon waves in the North Atlantic during a voyage from Liverpool to Boston. The results of his observations have not yet been published.

OBSERVATIONS IN THE UNITED STATES.

Observations at Oswego, N. Y.—During the season of 1884 observations were made by Lieut. Col. Henry M. Robert, U. S. Corps of Engineers, at Oswego Harbor, New York, upon the height, velocity, and force of the waves at the western part of the west breakwater: "The height of the waves was determined by a level placed upon the high lake bank west of the shore and directed lakeward upon the incoming waves. The height determined was that attained by the wave when distant about 1,000 feet from the breakwater. The velocities

were determined by the time required for the wave to sweep along the shore arm of the breakwater."

Measurements of the force exerted by the waves were attempted by means of four dynamometers placed as follows: One 16 feet below the surface of the water; the second, 8 feet below; the third, at the water surface; and the fourth, 8 feet above it. The results of these observations are given in the Report of the Chief of Engineers, United States Army, for 1885, page 2279, and will be alluded to hereafter. No observations appear to have been taken on the height of the waves from hollow to crest, their wave lengths, or the depth of water in which they broke. These observations, although somewhat limited in number, are of interest, as they are believed to constitute the first successful attempt made in this country to apply the method of measuring wave force devised by Thomas Stevenson, although at Lewes, Del., in 1873, Capt. M. R. Brown, U. S. Corps of Engineers, had procured several dynamometers which he proposed to fasten to an iron pile in front of the breakwater. Before this could be accomplished a vessel ran into the pile and destroyed it, and the attempt to secure dynamometer readings at this point appears never to have been resumed.

Observations at Milwaukee, Wis.—Lieut. C. H. McKinstry, U. S. Corps of Engineers, took dynamometer readings at the harbor of refuge, Milwaukee Bay, Wisconsin, during three storms which occurred in February, April, and May, 1894. A detailed account of these observations is given in the Report of the Chief of Engineers, United States Army, 1894, page 2086. Two dynamometers were used in each case, the plates being located 6½ feet above the surface of the water during the first storm, and 2½ feet above during the second and third storms. Seven readings in all were obtained, the details concerning which will be given hereafter.

Observations at St. Augustine, Fla.—In 1890 and 1891, while engaged in constructing concrete groins on North Beach, St. Augustine, Fla., the writer carried on a series of observations upon waves immediately before and at the instant of breaking; this breaking being caused by uniform and gradually decreasing depth.

Observations were taken upon 283 waves in all, varying in height, at the instant of breaking, from 2 inches to 7 feet.

These observations embraced:

(*a*) The height of the wave just before breaking;

(*b*) Its velocity at this time;

(*c*) The relation of the wave height to the depth of water in which the wave broke;

(*d*) The relation of hollow and crest to the undisturbed surface level;

(*e*) The ratio of the wave height to the wave length; and

(*f*) The measurement of the maximum force developed.

Three specially devised dynamometers (see p. 151) were used in measuring the force developed by the wave, and 197 readings, on 105 different dates, were secured.

These readings were for waves varying in height from 2½ to 6 feet.

The results obtained from these observations will be discussed hereafter.

Observations on Lake Superior.—During the seasons of 1901–3 the writer secured over 2,000 observations in all upon shallow-water waves at five different localities on Lake Superior, most of them, however, were taken near the outer ends of the ship-canal piers at Duluth, Minn. The waves observed varied in height from 2 to 23 feet, and observations were taken to determine—

(*a*) The wave height;

(*b*) The length;

(*c*) The velocity;

(*d*) The relation of hollow and crest to the undisturbed water level;

(*e*) The depth in which waves broke;

(*f*) The effect of decreasing depth upon the wave velocity;

(*g*) The relation between wind velocity and wave velocity;

(*h*) The maximum wave force developed, as determined by dynamometer measurements;

(*i*) The character of wave impact;

(*j*) The height of waves due to "fetch;" and

(*k*) The reduction of height on entering a closed harbor.

(*l*) The static pressure due to passing waves.

CHAPTER III.

THEORY OF WAVE MOTION.

Deep-water waves.—Conditions governing wave motion. Equations relating to prolate cycloid. Relation of hollow and crest of wave to undisturbed water level. Wave velocity. Rate of decrease of orbit radii. Wave energy.

Shallow-water waves.—Character of wave motion in shallow water. Axes of orbits. Wave velocity. Wave energy.

Discussion of other characteristics.—Superposition of waves. Transmission of wave energy. Momentum of deep-water waves. Margin of stability.

The complex and confused character of waves during storms generally prohibits the direct application of theoretical formulæ to waves of this class, but observation and study of the simpler and more regular types of waves, to which the theoretical formulæ are applicable, will result in a better knowledge of the action of all classes of waves than can be attained without the aid of such formulæ. Although numerous theories of wave motion have from time to time been advanced, most of the older theories have proved to be fallacious, and the cycloidal or trochoidal theory is now generally acknowledged as representing most closely the simpler and more regular forms of wave motion.

A correct theory of wave motion must satisfy the following conditions:

(1) Dynamic equilibrium—i. e., the effective force acting on every particle to produce acceleration is the resultant of its weight and the pressure of the surrounding liquid.

(2) Continuity—i. e., a mass of water may change form during the passage of a wave, but its continuity must not be broken.

(3) Boundary—i. e., depth, character of the bottom, area of the surface of the liquid, and the pressure upon the same.

(4) Formation—i. e., the possibility of producing wave motion from still water by wind action, due consideration

being given to the viscosity of the liquid, and its condition as to molecular rotation.

The first three conditions are satisfied by the trochoidal theory, but the last is not, and it is therefore probable that actual waves, even of the most regular character, do not assume the precise form of the trochoid, although differing but little therefrom.

WAVES IN DEEP WATER.

The trochoidal theory of deep-water waves refers to a regular and uniform series of waves of equal dimensions, in water of unlimited area and depth, and subjected to uniform surface pressure, each particle revolving with uniform speed in a circular orbit in a vertical plane perpendicular to the wave crest, and making a complete revolution while the wave advances through a distance equal to the wave length. The character of the motion of the particles is shown graphically in fig. 1, page 16.

The theory of wave motion has been treated very thoroughly mathematically by Prof. G. B. Airy, Professor Rankine, Mr. Froude, Mr. Stokes, and others, and no attempt will be made here to follow their deductions step by step, but instead, only such of their conclusions will be given as bear upon the practical side of the subject, or may be compared with results obtained by observations upon actual waves.

The equations of the prolate cycloid, referred to the rectangular coordinate axes C D and C F, fig. 3, are:

$$x = R\theta - r \sin \theta, \qquad (1), \text{ and}$$

$$y = R - r \cos \theta, \qquad (2),$$

in which:

R=Radius of the rolling circle.
r=Radius of the tracing circle.
θ=Angle of revolution of the rolling circle about its center.
r_s=Value of r for surface orbit.

At any point of the trochoid corresponding to the angle θ, the inclination of the tangent to the horizontal, or of the normal to the vertical, is—

$$\tan \phi = \frac{r \sin \theta}{R - r \cos \theta}.$$

The slope is a maximum at the point of inflection—i. e., *above* the mid height of the wave, for which $\cos\theta = \frac{r}{R}$. CB (fig. 7) is then perpendicular to AB and the slope is—

$$\phi = \tan^{-1}\frac{r}{\sqrt{R^2-r^2}}.$$

Denoting the length of the wave from hollow to hollow by L, the wave height by h, and the area of the portion of the cross section of the wave lying above A B by A, we have—

$$A = 2\pi R\left(r_s - \frac{r_s^2}{2R}\right); \qquad (3).$$

The position of the center of gravity of the portion of the wave lying above A B is raised above the position which it occupies in an equivalent mass of water in an undisturbed state, a distance

$$q = \frac{r_s}{4}\cdot\frac{2R^2 - r_s^2}{2R^2 - Rr_s}; \qquad (4).$$

Elevation of wave crest.—To find what proportion of the wave height lies above the undisturbed water level, we have from fig. 3 and from equation (3),—

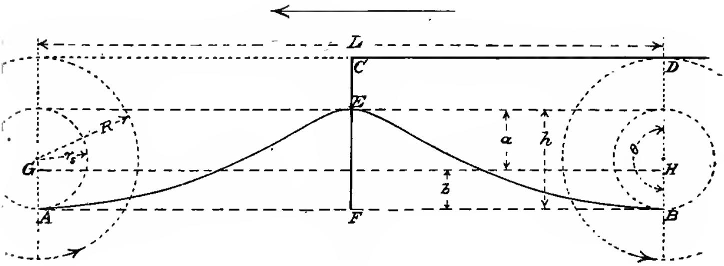


Fig. 3.

$$a+b=h=2r_s$$

$$bL = 2\pi R\left(r_s - \frac{r_s^2}{2R}\right); \qquad \text{in which}$$

a=height of wave crest above still-water level.

b=depth of wave hollow below still-water level.

Hence

$$b = \frac{2\pi R}{L}\left(r_s - \frac{r_s^2}{2R}\right) = r_s - \frac{\pi r_s^2}{L} = \frac{h}{2} - 0.7854\frac{h^2}{L};$$

$$a = h - \left(r_s - \frac{r_s^2}{2R}\right) = r_s + \frac{\pi r_s^2}{L} = \frac{h}{2} + 0.7854\frac{h^2}{L}; \qquad (5).$$

Velocity.—The velocity of a wave in deep water is given by the formula:

$$v=\sqrt{\frac{gL}{2\pi}}=\sqrt{gR}=\sqrt{5.123L}; \qquad (6),$$

g representing the acceleration due to gravity and the other quantities being as before.

From equation (6) it is seen that the wave velocity v is practically the same as that which a heavy body would acquire by falling through a distance equal to 8 per cent of the wave length.

Lieutenant Paris suggested, from an analysis of his own observations, that the speed of the waves is proportional to the square root of the speed of the wind, or

$$v^2=\frac{\text{Speed of wind.}}{0.022}$$

As explained in Chapter V, it is in most cases difficult to utilize such a relation, should it be proved to exist, for the reason that the wind velocity and direction may differ considerably at the two extremities of the "fetch" considered.

Law of decrease of orbit radii.—As the distance below the surface of the water increases in arithmetical progression the radii of the orbits decrease in geometrical progression, the law of their decrease being expressed as follows:

$$r=r_s\varepsilon^{\frac{-2\pi d'}{L}}; \qquad (7)$$

in which r, r_s, and L are as before given; ε is the base of the Naperian system of logarithms, and d' is the depth measured from the center of the surface orbit.

For convenience in computation formula (7) may be written—

$$\log r=\log r_s-2.72875\frac{d'}{L}; \qquad (8).$$

The rate of decrease of the radii is given in Table I, page 39, and is shown graphically by the curve EF, Pl. V.

TABLE I.—*Showing rate of decrease of orbits of particles of deep-sea waves as the depth below the mean position of the surface-orbit increases, the radius of the surface orbit being assumed as unity.*

$\frac{d'}{L}$	r	$\frac{d'}{L}$	r	$\frac{d'}{L}$	r	$\frac{d'}{L}$	r
0.00	1.000	0.12	0.471	0.48	0.049	0.92	0.0031
.01	.939	.14	.415	.52	.038	.96	.0024
.02	.883	·16	.367	.56	.029	1.00	.0019
.03	.828	.18	.323	.60	.023	1.50	.000081
.04	.779	.20	.285	.64	.018	2.00	.0000035
.05	.730	.24	.221	.68	.014	....	....
.06	.684	.28	.172	.72	.011	....	....
.07	.644	.32	.134	.76	.0084	....	....
.08	.605	.36	.104	.80	.0066	....	....
.09	.568	.40	.081	.84	.0051	....	....
.10	.533	0.44	.063	.88	.0040	....	....

Wave energy.—The energy in a wave consists of the kinetic energy, due to the revolution of the particles in their orbits, and the potential energy due to the elevation of the center of gravity of the mass of water composing a wave above its position in the corresponding mass at rest.

The kinetic energy of a particle is—

$$\frac{m\phi^2 r^2}{2} = mg\frac{r^2}{2\,R},$$

ϕ here representing the angular velocity of the particle in its orbit.

The mean elevation of the particles in any trochoidal surface throughout a wave above their corresponding positions in still water is $\frac{r^2}{2R}$, which for surface particles becomes $\frac{r_s^2}{2R}$. The mean potential energy of each particle during one revolution is $mg\frac{r^2}{2R}$, m representing the mass of the particle.

It will be noticed that the expressions for the kinetic and potential energy of a particle are identical. It therefore follows that the total energy of a wave is one-half kinetic and one-half potential.

Considering the kinetic energy due to the motion of all particles between the surface, and the depth corresponding to r, and integrating between proper limits, there results for the

kinetic energy in a wave of length L, height h, and of unit breadth, between the surface and the depth corresponding to r

$$E_k' = \frac{w\pi}{R}\left[\frac{r^4 - r_s^4}{4} - \frac{R^2}{2}(r^2 - r_s^2)\right]; \qquad (9),$$

which, when r becomes zero, gives for the total kinetic energy of the wave

$$E_k = \frac{w\pi r_s^2}{4R}(2R^2 - r_s^2) = \frac{wLh^2}{16}\left(1 - \frac{\pi^2 h^2}{2L^2}\right); \qquad (10),$$

w representing the weight of a cubic foot of water. The total potential energy in the wave may be obtained by multiplying together the second terms of equations (3) and (4) and multiplying the result by w, the weight of a cubic foot of water.

$$E_p = 2w\pi R\left(r_s - \frac{r_s^2}{2R}\right) \times \frac{r_s^4}{4} \cdot \frac{2R^2 - r_s^2}{2R^2 - Rr_s}$$

$$= \frac{w\pi r_s^2}{4R}(2R^2 - r_s^2) = \frac{wLh^2}{16}\left(1 - \frac{\pi^2 h^2}{2L^2}\right); \qquad (11).$$

The last equation, as should be the case, is identical with equation (10), which expresses the kinetic energy in the wave.

The total wave energy, both kinetic and potential, for a wave of length L, height h, and unit breadth is, therefore—

$$E = E_k + E_p = \frac{wLh^2}{8}\left(1 - \frac{\pi^2 h^2}{2L^2}\right) = 8Lh^2\left(1 - 4.935\frac{h^2}{L^2}\right); \quad (12),$$

the weight of a cubic foot of sea water being taken as 64 pounds.

It may be shown mathematically that, during a single complete wave period, an amount of energy equal to half of the total energy of the wave is transmitted forward with the wave form.

From equation (12) by assuming suitable values for the wave height h and the wave length L, the following table has been prepared:

TABLE II.—*Total energy of deep-water waves in foot-tons per foot of wave crest.*

$\frac{h}{L}$	Wave length, in feet.										
	50	100	150	200	250	300	400	500	600	700	800
0.02	0.2	1.6	5.4	12.8	24.9	43.1	102.2	199.6	344.9	547.7	818
.04	.8	6.3	21.4	50.8	99.2	171.4	406.4	793.7	1,371.5	2,177.9	3,251
.06	1.8	14.1	47.7	113.1	221.0	381.9	905.2	1,767.9	3,055.0	4,851.3	7,242
.08	3.1	24.8	83.7	198.3	387.4	669.4	1,586.6	3,098.9	5,354.9	8,503.3	......
.10	4.7	38.0	128.3	304.2	594.1	1,026.6	2,433.5	4,753.0	8,213.2	....	......
.12	6.7	53.5	180.6	428.0	836.0	1,444.6	3,424.3	6,688.1	....	....	......
.14	8.8	70.8	239.0	566.5	1,106.5	1,912.1	4,532.4	....	....	....	......
.16	11.2	89.5	302.0	715.7	1,397.9	2,415.6	....	....	....	....	......
.18	13.6	108.9	367.5	871.0	1,701.2	2,939.7	....	....	....	....	......
.20	16.0	128.4	433.4	1,027.0	2,006.5	3,467.2	....	....	....	....	......

WAVES IN SHALLOW WATER.

The class of waves known technically as "shallow-water waves" are those which are propagated in water of less depth than half a wave length. A wave of this class differs from a deep-water wave in that the orbits of its particles are ellipses instead of circles, the eccentricity of the ellipses depending upon the ratio of the wave length to the depth of the water.

For a given wave length the eccentricity decreases with an increase of depth, until at depths greater than half a wave length the ellipses are scarcely distinguishable from circles, while in very shallow water they tend to approach right lines.

The orbits decrease in size below the surface, their focal distance remaining constant, the vertical axes therefore decreasing more rapidly relatively than the horizontal, until, at the bottom, were the latter horizontal and frictionless, the vertical axes would become zero, and the particles would move in horizontal straight lines of length equal to the focal distance of the elliptical orbits of the upper particles.

Each particle revolves about the center of the ellipse with an angular velocity which is not constant, but is greater in the vicinity of the crest and trough, and less at mid height than if the orbit were a circle.

If a circle be drawn with a radius equal to the semi-major (horizontal) axis of the ellipse, and having the same center, the velocity of a particle in its elliptical orbit is such that a point vertically above it on the circle moves with constant angular velocity. (See Fig. 10, p. 97.)

Taking the origin at the orbit center of a crest particle,

and remembering that θ=ACm′ (fig. 4) the equations of the reduced trochoid are—

$$x = R\theta - a \sin\theta$$
$$y = b \cos\theta$$

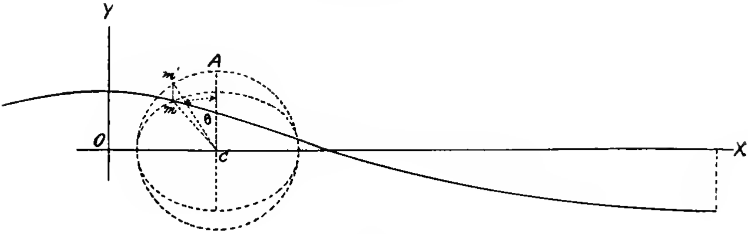


Fig. 4.

Let a' and b'=the semi-major and semi-minor axes of the elliptical orbits at the depth d'; a_s and b_s=the semi-major and semi-minor axes of the elliptical surface orbits; d'=depth from center of surface orbits to center of orbits whose semi-axes are a' and b'; d_o=depth from center of surface orbits to bottom, and L=the wave length.

For the surface orbits—

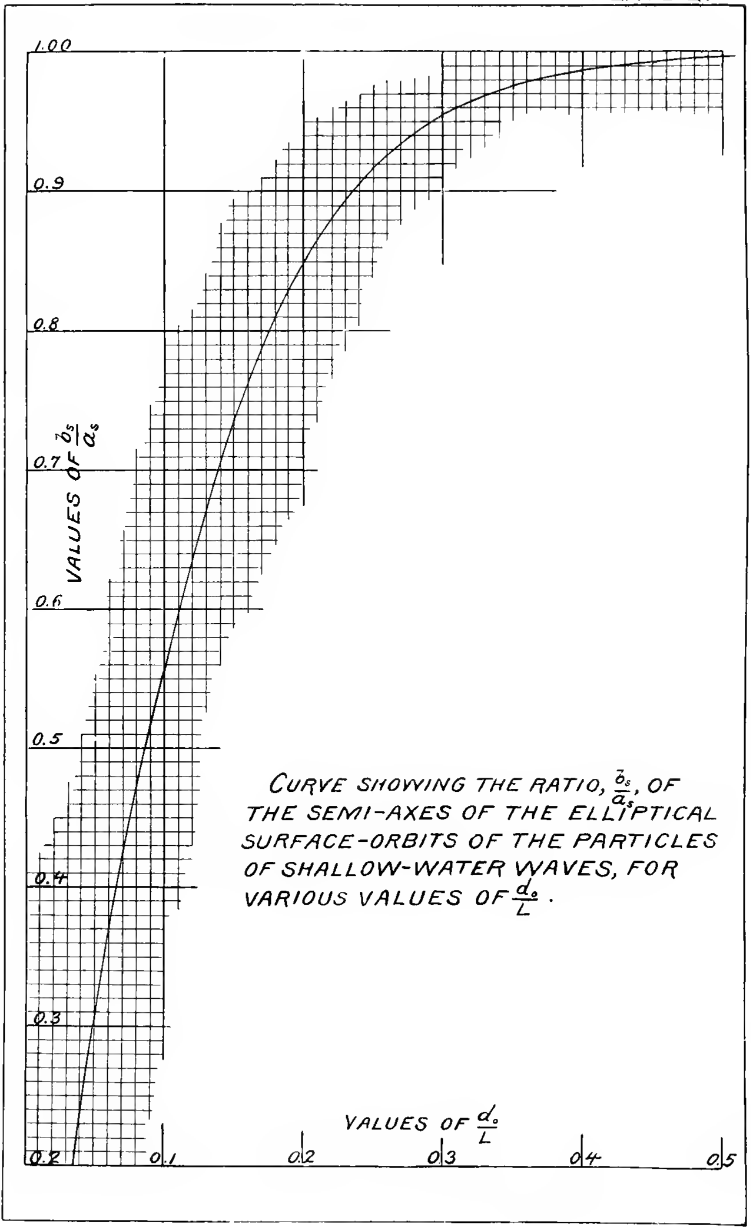
$$a_s = b_s \frac{\varepsilon^{\frac{4\pi d_o}{L}} + 1}{\varepsilon^{\frac{4\pi d_o}{L}} - 1}; \qquad (13).$$

The following table gives the ratio of the axes of the surface orbits of shallow-water waves for various lengths of waves and depths of water. This ratio is shown graphically for values of $\frac{d_o}{L}$ between 0 and 0.5 in Pl. I.

TABLE III.

Depth in per cent of wave length. $\frac{d_o}{L}$	Ratio of axes of surface orbits. $\frac{b_s}{a_s}$	Depth in per cent of wave length. $\frac{d_o}{L}$	Ratio of axes of surface orbits. $\frac{b_s}{a_s}$
0.05	0.3043	0.55	0.9980
.10	.5569	.60	.9989
.15	.7366	.65	.9994
.20	.8500	.70	.9997
.25	.9171	.75	.9998
.30	.9549	.80	.99991
.35	.9757	.85	.99995
.40	.9869	.90	.99998
.45	.9930	.95	.999987
.50	.9963	1.00	.999993

PLATE I.


1.00
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
VALUES OF $\frac{b_s}{a_s}$
0.1
0.2
0.3
0.4
0.5
VALUES OF $\frac{d_o}{L}$
CURVE SHOWING THE RATIO, $\frac{b_s}{a_s}$, OF THE SEMI-AXES OF THE ELLIPTICAL SURFACE-ORBITS OF THE PARTICLES OF SHALLOW-WATER WAVES, FOR VARIOUS VALUES OF $\frac{d_o}{L}$.

From this table it will be noticed that there is practically no difference between the deep-sea and shallow-water wave when the depth of the water is not less than half the wave length.

The ratio $\frac{b_s}{a_s}$ of the semi axes of the surface orbits enters as a factor in the expressions for the velocity, period, etc., of the shallow-water wave. As the depth of the water increases this ratio approaches unity, and when the depth is great, the formulæ for the shallow-water wave become identical with those previously deduced for the deep-water wave.

The semi axes a' and b' of the elliptical orbit whose center is at the depth d' below the centers of the surface orbits, are given by the following equations:

$$b' = b_s \frac{\varepsilon^{\frac{2\pi(d_o-d')}{L}} - \varepsilon^{\frac{-2\pi(d_o-d')}{L}}}{\varepsilon^{\frac{2\pi d_o}{L}} - \varepsilon^{\frac{-2\pi d_o}{L}}}; \qquad (14).$$

$$a' = b_s \frac{\varepsilon^{\frac{2\pi(d_o-d')}{L}} + \varepsilon^{\frac{-2\pi(d_o-d')}{L}}}{\varepsilon^{\frac{2\pi d_o}{L}} - \varepsilon^{\frac{-2\pi d_o}{L}}}; \qquad (15).$$

These equations are readily solved when it is remembered that $\varepsilon^{\frac{2\pi(d_o-d')}{L}}$ is the number whose common logarithm is

$$\frac{2\pi(d_o-d')}{L} \times 0.434294.$$

Comparing the square of equation (13) with the difference of the squares of equations (14) and (15), we have

$$a'^2 - b'^2 = a_s^2 - b_s^2, \text{ equivalent to } 2\sqrt{a'^2 - b'^2} = 2\sqrt{a_s^2 - b_s^2}.$$

The two members of this last equation represent the focal distances of the orbits of a particle at any depth d', and at the surface, respectively, and show that for any particular wave the focal distance of the elliptical orbits of the particles is the same at all depths.

By means of equations (14) and (15) the following table has been computed, showing the rate of decrease of the two axes of the elliptical orbits, at the surface, at mid depth, and on the bottom, for different ratios between the wave length L and the depth d_o from the center of the orbits of the surface particles to the bottom; the minor axis of the orbit at the surface, which is equal to the wave height h, being taken as unity.

TABLE IV.

$\frac{d_0}{L}$	Ratio of axes of elliptical orbits at—					
	Surface.		Mid depth.		Bottom.	
	Minor axis.	Major axis.	Minor axis.	Major axis.	Minor axis.	Major axis.
0.05	1.0000	3.2862	0.4939	3.1707	0	3.1319
.10	1.0000	1.7957	.4762	1.5657	0	1.4914
.15	1.0000	1.3576	.4491	1.0228	0	.9193
.20	1.0000	1.1765	.4153	.7458	0	.6194
.25	1.0000	1.0904	.3774	.5756	0	.4345
.30	1.0000	1.0472	.3383	.4594	0	.3108
.35	1.0000	1.0249	.2998	.3745	0	.2246
.40	1.0000	1.0133	.2633	.3097	0	.1631
.45	1.0000	1.0070	.2296	.2585	0	.1187
.50	1.0000	1.0037	.1993	.2173	0	.0866

The rate of decrease of the orbit axes for a particular wave is shown graphically on Pl. V.

Velocity of shallow-water waves.—The velocity of the shallow-water wave is less than that of the deep-water wave, and is as follows:

$$v=\sqrt{\frac{b_s}{a_s}.\frac{gL}{2\pi}}=\sqrt{\frac{b_s}{a_s}\ gR}=\sqrt{5.123\,\frac{b_s}{a_s}\,L}; \qquad (16).$$

The following table shows the velocity of the shallow-water wave in per cent of the velocity of the deep-water wave of the same wave length for various depths of water:

TABLE V.

Depth below center of surface orbits, in per cent of wave length. $\frac{d_0}{L}$	Velocity of shallow-water wave, in per cent of velocity of deep-water wave of same wave length.	Depth below center of surface orbits, in per cent of wave length. $\frac{d_0}{L}$	Velocity of shallow-water wave, in per cent of velocity of deep-water wave of same wave length.
0.05	0.5516	0.55	0.9990
.10	.7464	.60	.9995
.15	.8583	.65	.9997
.20	.9219	.70	.99985
.25	.9576	.75	.99990
.30	.9772	.80	.99995
.35	.9878	.85	.99997
.40	.9935	.90	.99999
.45	.9965	.95	.999995
.50	.9982	1.00	.999999

Mean elevation of particles.—The mean elevation of the particles of the approximate trochoidal surfaces throughout the wave length above their corresponding still-water positions is $\frac{a' b'}{2R}$, which, as the depth becomes greater in proportion to the wave length, approaches more nearly to $\frac{r^2}{2R}$, the value previously found for the deep-water wave of the same wave length.

Energy of shallow-water wave.—The energy of the shallow-water wave is slightly less than that of the deep-water wave of equal height and length, the difference in energy depending on the depth of the water. As in the case of the deep-water wave, the energy is half kinetic and half potential, the latter being transmitted onward with the wave form.

The potential energy of the shallow-water wave may be deduced as follows:

From the geometrical construction of the surface profile of a shallow-water wave, as shown in fig. 5, it will be seen that the difference *e e'* between this curve and the corresponding

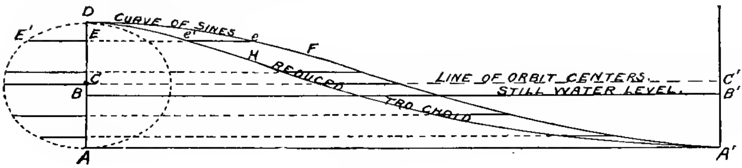


Fig. 5.

curve of sines at any height A E is equal to the corresponding horizontal semi chord E E' of the elliptical orbit of a surface particle. The difference, therefore, between the two areas A D F A' and A D H A' is equal to $\frac{\pi}{2} a_s b_s$, the area A D E' of the semi ellipse.

The area A D F A' is from the symmetry of the curve D F A' equal to half the product of the height AD and length A A' $= \frac{hL}{4}$.

Considering the entire wave length L, the area of the cross section of the reduced trochoidal wave is—

$$A = \frac{hL}{2} - \pi a_s b_s.$$

The height of the center of gravity of this area above a horizontal plane through the wave hollow is—

$$k'=\frac{h}{2}\left(\frac{3L-2\pi h}{4L-2\pi h}\right)-\frac{hL}{4}\left(\frac{2\pi a_s-\pi h}{(4L-2\pi h)(L-\pi a_s)}\right).$$

The height of the center of gravity of the identical volume of water above the same datum plane, before wave motion commences, is—

$$k=\frac{Lh-\pi a_s h}{4L}.$$

The "lift" of the center of gravity, due to wave motion, is therefore $=k'-k$.

Considering the volume included between two parallel vertical planes, perpendicular to the wave crest, and at unit distance apart, it will be seen that the total potential energy of the wave will be represented by the weight of the fluid lying above a horizontal plane through the wave hollow, multiplied by the distance, $k'-k$, through which its center of gravity has been raised by wave motion, or—

$$E_p=wA(k'-k);$$

w representing the weight of a cubic foot of water—taken as 64 pounds for salt water and 62.4 pounds for fresh water.

Substituting for w, A, k' and k their proper values, we have—

$$E_p=\frac{wh}{2}\left(\frac{2hL^3-\pi h^2L^2-4\pi^2a_s^2hL+2\pi^3a_s^2h^2}{4L(4L-2\pi h)}\right);$$

$$=\frac{wLh^2}{16}\left(1-19.74\frac{a_s^2}{L^2}\right); \qquad (17).$$

Remembering that the kinetic and potential energy of the wave are equal, and that for salt water w=64 pounds, we have for the total energy of ocean waves in shallow water—

$$E_{k+p}=8Lh^2\left(1-19.74\frac{a_s^2}{L^2}\right); \qquad (17A).$$

When the depth is great $a_s=b_s=\frac{h}{2}$, and equation (17A) becomes identical with equation (12), as it should.

To aid in solving equations (17) and (17A), the following table has been prepared, in which the quantities in the third and sixth columns are the numerical coefficients to be used with $\frac{h^2}{L^2}$ when substituted in the last term (in parenthesis) equations (17) and (17A).

TABLE VI.—*Value of the semi-major axis of a surface particle of a shallow-water wave, in terms of the wave height, for different values of* $\frac{d_o}{L}$; *with corresponding value of the numerical coefficient of the second term, in parenthesis, equations (17) and (17A).*

$\frac{d_o}{L}$	$a_s=$	$19.74\frac{a_s^2}{L^2}=$	$\frac{d_o}{L}$	$a_s=$	$19.74\frac{a_s^2}{L^2}=$
0.10	$h\div1.10$	$16.31\times\frac{h^2}{L^2}$	0.22	$h\div1.76$	$6.37\times\frac{h^2}{L^2}$
.11	$h\div1.19$	$13.90\times\frac{h^2}{L^2}$	.23	$h\div1.79$	$6.17\times\frac{h^2}{L^2}$
.12	$h\div1.27$	$12.26\times\frac{h^2}{L^2}$	.24	$h\div1.81$	$6.02\times\frac{h^2}{L^2}$
.13	$h\div1.34$	$10.97\times\frac{h^2}{L^2}$	.25	$h\div1.83$	$5.89\times\frac{h^2}{L^2}$
.14	$h\div1.42$	$9.77\times\frac{h^2}{L^2}$	.26	$h\div1.85$	$5.77\times\frac{h^2}{L^2}$
.15	$h\div1.47$	$9.14\times\frac{h^2}{L^2}$	.27	$h\div1.87$	$5.64\times\frac{h^2}{L^2}$
.16	$h\div1.53$	$8.44\times\frac{h^2}{L^2}$	.28	$h\div1.88$	$5.56\times\frac{h^2}{L^2}$
.17	$h\div1.57$	$8.02\times\frac{h^2}{L^2}$	.29	$h\div1.90$	$5.47\times\frac{h^2}{L^2}$
.18	$h\div1.62$	$7.53\times\frac{h^2}{L^2}$	.30	$h\div1.91$	$5.41\times\frac{h^2}{L^2}$
.19	$h\div1.66$	$7.15\times\frac{h^2}{L^2}$	.31	$h\div1.92$	$5.36\times\frac{h^2}{L^2}$
.20	$h\div1.70$	$6.83\times\frac{h^2}{L^2}$	.32	$h\div1.93$	$5.31\times\frac{h^2}{L^2}$
.21	$h\div1.73$	$6.60\times\frac{h^2}{L^2}$	.33	$h\div1.94$	$5.25\times\frac{h^2}{L^2}$

Taking the extreme case in which the depth measured from the line of centers of the orbits of surface particles is but one-tenth of the wave length, and assuming that the wave height is one-tenth of the wave length, we find by making the proper substitutions in equation (17A), that the total theoretical energy of the wave, both kinetic and potential, is only about 11 per cent less than that of a deep-water wave of the same height and wave length.

For the more usual case, in which $\frac{d_o}{L}=.14$, and $\frac{h}{L}=0.067$, the total energy of the shallow-water wave is but about 2 per cent less than that of the deep-water wave of the same height and wave length.

When $\frac{d_o}{L}$ is greater than 0.25 and $\frac{h}{L}$ less than 0.1, the energy of the shallow-water wave will differ from that of the deep-water wave of the same height and wave length by less than 1 per cent, and Table II may be used for both classes of salt-water waves. For all fresh-water waves the quantities given in the table must be reduced by 2½ per cent.

It will be shown hereafter that the actual profile of a shallow-water wave is usually sharper in the vicinity of the crest, and broader and flatter in the vicinity of the hollow than is the case with the theoretical shallow-water wave. This fact taken alone would indicate that the energy given by equation (17) was somewhat in excess of the energy of an actual shallow-water wave of the same height and wave length. Observation, however, shows that the center of gravity of the actual shallow-water wave is lifted higher than theory would indicate, and the same is true of the centers of surface orbits; which facts alone considered would indicate a greater energy for the actual shallow-water wave than for the theoretical wave. Consequently the departures from the assumed form tend to neutralize one another in the actual shallow-water wave, and when these waves are regular it is probable that equation (17) gives results which represent their energy approximately correctly.

Velocity of long waves.—Professor Kelland has shown that in the case of a uniform canal, the cross-section of which is of any form, the velocity of "long waves" (i. e., waves whose length is very great in comparison with the depth) is—

$$v = \sqrt{g\frac{A}{b}}, \qquad (16A),$$

in which A = the cross-sectional area, and b = the breadth of the fluid.

Virtual depth.—Imagine two fixed vertical planes, the one passing through the wave crest, and the other through the trough. The water during wave motion is passing through these planes with a velocity greatest at the surface, and decreasing with the depth. If, instead of varying in this manner, the velocity were constant and equal to that at the surface, there would be some depth, such that with this constant velocity throughout, the same quantity of water would be discharged as passes with the actual variable velocity through the plane of infinite depth. If d_1 and d_2 represent such a depth for the crest plane and trough plane, respectively, we may imagine some mean depth D such that $D = \frac{d_1 + d_2}{2}$.

This mean depth D, is known as the virtual depth of the wave, and is a measure of disturbance of the water below the

surface. Whatever may be the nature of any particular wave, the virtual depth will always control its speed of propagation.

For the ordinary trochoidal wave D = R; for the shallow-water wave $D = \frac{b_s}{a_s} R$, and for the solitary wave of translation $D = d_c$, or the virtual depth is equal to the total depth of water at the crest.

Superposition of waves.—If the height of a deep-water wave is so small in comparison with its length that the profile does not differ materially from a curve of sines it can be shown mathematically that when it is reflected from a vertical wall after normal impact and meets similar oncoming waves, the particles where the compound profile cuts the line of orbit centers are fixed and at rest, while midway between these points they vibrate vertically, forming trough and crest, alternately, the extent of the vibration above and below the line of orbit centers at these points being—

$$\pm 2r \cos \frac{vt'}{R};$$

t' representing the time which has elapsed since the crest was on the axis of Y. The axis of X is taken along the line of orbit centers, and the values of x for the troughs and crests of the compound profile is $\frac{nL}{4}$, n being an even number. The motion of the particles at intermediate points is along straight lines of length and obliquity depending on their positions in the wave profile. Such motion constitutes a "chopping sea."

Groups of waves.—A group of waves separated from another group by short intervals of still water, may be formed under certain conditions from a series of approximately trochoidal waves of equal height and very nearly the same length and velocity, traveling in the same direction.

Investigation of the character of the motion shows that there may be an indefinite number of successive groups, separated by short stretches of still water; each group being composed of a series of waves, the height at the center of the group being double that of the simple wave, and decreasing down to still water at the two extremities of the group.

Professor Stokes has shown that the velocity of propagation of the group as a whole is half that of the individual wave, consequently each individual wave, beginning at the rear, travels forward, increasing in height to the center of the group, thence decreasing gradually until it merges into still water in front of the group.

If l denotes the actual length of the group (the distance between consecutive still-water spaces at any instant), the individual waves will travel a distance equal to $2\ l$ to reach similar recurring phases.

TRANSMISSION OF WAVE ENERGY.

It has been stated that in equal oscillatory deep-water waves one-half of the total energy of the wave is transmitted forward with the wave form. The energy thus transmitted is the potential energy of the wave, which is measured by the elevation of the center of gravity of the mass of water composing the wave above the position which it occupied in still water. The kinetic energy undergoes no change, since under the conditions assumed the particles continue to describe circles of the same radius, with unchanged angular velocity.

Energy of waves in groups.[a]—Suppose that in a trough of water originally at rest, one end of the trough is flexible and capable of vibrating with accompanying change of shape, so as to follow the motion of trochoidal columns, the other end of the trough being at an infinite distance. Let the flexible end of the trough vibrate in any given period, and let energy $\frac{E'}{2}$ from without be impressed upon it at each forward swing.

Undulations will thus be formed whose length will depend on the period of vibration of the flexible end of the trough, and the height of successive waves will gradually increase, until finally the height will be that due to energy E', double the energy of vibration of the flexible end of the trough.

If n denotes the number of waves in a group, and if the waves are numbered in the order of production, the energy of the first wave will always be equal to $\frac{E'}{2^n}$. For the

[a] From article on "Water Waves," by Naval Constructor R. Gatewood, U. S. Navy.

middle wave (if n is an odd number) the energy is $\frac{E'}{2}$, and for the nth wave it is $\frac{E'}{2^n}(2^n-1)$.

The distribution of energy for a limited number of waves is as follows:

Energy of—	Total number of waves.						
	1.	2.	3.	4.	5.	6.	7.
First wave	$\frac{1}{2}$ E′	$\frac{1}{4}$ E′	$\frac{1}{8}$ E′	$\frac{1}{16}$ E′	$\frac{1}{32}$ E′	$\frac{1}{64}$ E′	$\frac{1}{128}$ E′
Second wave	......	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{5}{16}$	$\frac{6}{32}$	$\frac{7}{64}$	$\frac{8}{128}$
Third wave	......	......	$\frac{7}{8}$	$\frac{11}{16}$	$\frac{1}{2}$	$\frac{22}{64}$	$\frac{29}{128}$
Fourth wave	......	......	......	$\frac{15}{16}$	$\frac{26}{32}$	$\frac{42}{64}$	$\frac{1}{2}$
Fifth wave	......	......	......	......	$\frac{31}{32}$	$\frac{57}{64}$	$\frac{99}{128}$
Sixth wave	......	......	......	......	......	$\frac{63}{64}$	$\frac{120}{128}$
Seventh wave	......	......	......	......	......	......	$\frac{127}{128}$
Total energy of group	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$

The difference of energy between the middle wave and either of two waves equidistant from the middle is the same, the energy of one being greater and of the other less than that of the middle wave, consequently it follows that the total energy is being transmitted at the speed of the middle wave, i. e., at half the speed of the individual waves.

Transmission of energy by solitary wave of translation, or positive wave.—The energy of the positive wave is partly potential and partly kinetic. As this wave leaves still water, at the original level, behind it when it passes, it is obvious that the total energy is transmitted with the wave and will be delivered at any point diminished only by the almost insensible effect of fluid friction.

When the depth decreases gradually and uniformly the ordinary shallow-water wave tends to assume the characteristics of the positive wave, and may therefore deliver at any point an amount of energy approximating the total energy of the wave.

Momentum of deep-water waves.—It may be shown mathematically that the wave taken as a whole has no momentum.

Considering only the first quarter of the wave length (between the values $\theta=0$ and $\theta=90°$, fig. 3) the horizontal momentum of a layer of infinitesimal thickness is

$$\frac{\mathrm{W}}{g}nzr\omega,$$

in which w = the weight of unit volume; n = the value of "virtual gravity" (for explanation of "virtual gravity," see Chapter IV); z = the normal thickness corresponding to n (nz = a constant); r = the orbit-radius of a particle, and ω the angular velocity of the particle in its orbit.

For the second quarter of the wave (between the values $\theta = 90°$ and $\theta = 180°$) the expression is identically the same, but with a contrary sign.

The total horizontal momentum of the first quarter of the wave, i. e., for the full depth, is—

$$-\frac{\mathrm{w}}{g}\omega \mathrm{R}^2 r_s\left(1-\frac{r_s^{\,2}}{3\mathrm{R}^2}\right) \qquad (18).$$

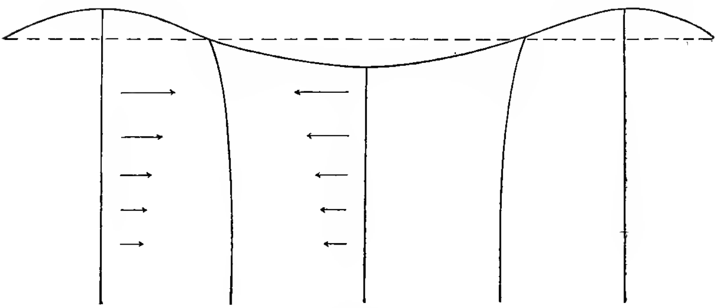


Fig. 6.

The last term in parenthesis is small in most actual cases and can usually be neglected. If this is done equation (18) becomes—

$$-\frac{\mathrm{w}}{g}\omega \mathrm{R}^2 r_s = -\frac{\mathrm{w}}{2g}\mathrm{Rv}h \qquad (18\mathrm{A}).$$

If, as in the figure above (i. e. fig. 6), wave columns are drawn corresponding to $\theta = 0°, 90°, 180°$, and $270°$, the wave will be divided into four portions as regards horizontal momentum, the values of which in adjacent portions will be equal, but opposed in direction about the wave columns corresponding to $\theta = 90°$ and $\theta = 270°$.

In like manner the wave may be divided into the same number of portions as regards vertical momentum, and similar expressions obtained.

Margin of stability.—In all waves the margin of stability varies along the wave profile, being greatest at the trough and decreasing rapidly toward the crest, the greater the value of $\frac{h}{L}$ the smaller the margin of stability at the crest.

At the crest of high waves[a], "the surface tension, which for waves of average height may be almost neglected as one of the forces in equilibrium, assumes an unwonted importance, and any interference with this surface tension may completely neutralize the very slight margin of stability remaining, thus causing the wave to break. This is taken advantage of to produce the well-known calming effect of oil on a heavy sea. Experience has shown that the surface tension of different oils in contact with air varies from one-third to one-half that of water.

"If, therefore, we can cover a steep wave with a thin film of oil, the surface tension is rapidly and materially reduced, with the result of breaking the crest of the wave and thus seriously interfering with the surface stratification. The succeeding waves are lower, both on account of the energy dissipated, and because of the broken water, which is now freed from the action of central forces and acts only under gravity, thus disturbing the conditions of stratification and helping to depress the crest of the next wave".

This "applies only to the inherent tendency of the waves to break. The direct effect of the wind in causing breaking waves is otherwise influenced by the pressure of the oil film."

Oils possessing great cohesion, and which are therefore capable of forming a thin, strong film are more efficient for this purpose than those like crude petroleum, which possess less cohesive strength.

It has been proposed to use oil in another way to calm the waves when a heavy sea is breaking over a breakwater. This is, to pour large quantities of oil on the waves, which, refusing to mix with water, tends to collect immediately in the troughs, being acted on by gravity only and not by the central forces of the wave particles. Its weight would then disturb the pressure equilibrium, interfere with the stratification, and cause the waves to become lower.

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[a] "Water Waves" by Naval Constructor R. Gatewood, U. S. Navy.

Effect of sloping sides upon stability.—In considering waves in a canal with sloping sides it has been found, mathematically that wave motion is stable when the sides of the canal are inclined at an angle of $\frac{\pi}{4}$ to the horizon, and unstable when they are inclined at an angle $\frac{\pi}{6}$. The theoretical limit of stability must therefore lie somewhere between these values.

CHAPTER IV.

WAVE FORM AND PRESSURE UPON THE PARTICLES.

Trochoidal theory involves molecular rotation. Professor Stokes' investigations based upon an irrotational fluid. Trochoidal wave form most stable. Displacement of particles during wave motion and pressure upon same. Difference between profiles of waves in deep and in shallow water. Theoretical dimensions of orbits of particles in a shallow-water wave at the point of breaking. Comparison of theoretical and photographed profiles of shallow-water waves.

Deep-water waves.—It has previously been stated that the trochoidal theory of wave motion does not satisfy the condition of "formation," i. e., can not be generated from water at rest by the action of observed forces. A perfect fluid when at rest is irrotational, and if fluid motion is once irrotational it remains so always, consequently no wave motion involving molecular rotation can be generated from a perfect fluid at rest. The trochoidal theory of wave motion involves molecular rotation, the value of which for the deep-water wave is—

$$-\omega\frac{r^2}{R^2-r^2};$$

from which it will be seen that it increases from the bottom up, and very rapidly on approaching the surface.

No wave approximating the form of the common cycloid can be formed in nature, for in such a case, r being equal to R, the expression for the molecular rotation would become equal to infinity, an absurd supposition.

For the reasons just stated it is not probable that even in the most regular system of waves there exist, except through accident, waves which are of the precise form of the trochoid. Yet when the wave height is less than one-tenth of the wave length, which, as will be shown hereafter, is almost invariably the case, the trochoidal wave may be regarded as practically irrotational and of approximately the same form as the wave produced by natural forces from water at rest.

Professor Stokes has developed a refinement of the trochoidal theory based upon the condition that the motion must be susceptible of being produced from water at rest, and therefore involving no molecular rotation.

According to this theory the particles describe circular orbits about centers which move forward with the velocity $v\frac{r^2}{R^2}$; thus combining oscillatory motion with a slow motion of translation in the direction of wave travel. When the depth of the fluid is great, this progressive motion decreases rapidly as the depth of the particles considered increases.

In accordance with the preceding assumption, Professor Stokes investigated mathematically the motion of oscillatory waves with great elaborateness, and showed that the equation of the wave profile in deep water agreed with a trochoid to the third order, but that this was no longer true when carried to the fourth order, for he then showed that the wave lies a little above the trochoid at the trough and crest and a little below it in the shoulders—that is, the troughs are shallower and flatter for an equal height of wave, the difference in form of the elevated and depressed portions of the fluid being more conspicuous in shallow than in deep water.

For deep-water waves the deviation from the true trochoidal form for a wave height equal to one-tenth of the wave length is about 2 per cent, varying for any given wave length, as the cube of the wave height.

Under the assumption of "irrotational motion," Stokes showed that the steepest possible wave has a sharp angle of 120° at the crest.

The profile has also been investigated and traced for the neighborhood of the crest by Michell, who found that the extreme value of the wave height $h = .142$ L, and that the corresponding wave velocity is greater than in the case of infinitely small height in the ratio of $\sqrt{2}$:1.

Although Professor Stokes' treatment of the subject may represent actual conditions slightly more accurately than the ordinary trochoidal theory, yet it is more complicated, and for ordinary waves differs so little from the simpler theory that the latter is usually adopted as a working hypothesis.

When a wind of increasing velocity begins to act upon a body of water previously at rest, very confused systems of

oscillatory motion are produced, few of the forms being capable of indefinite propagation. Any observer who endeavors during the earlier stages of a storm to follow with his eye a particular wave will find that after traveling a certain distance it will gradually disappear, or become so confused with others that it can no longer be distinguished as an individual wave, while new waves will appear, themselves to undergo the same transformations of form. This confusion of forms indicates that the waves produced at this stage are not sufficiently stable to be propagated for any considerable distance without change of form. Trochoidal motion appears to be the only form of wave motion possessing sufficient stability to withstand successfully any considerable extraneous disturbance.

In consequence, when the less stable forms of waves come in contact with, or under the influence of, the more stable forms, the former are gradually destroyed, often contributing energy to the more stable systems. By a continuation of this process, involving the principle of "the survival of the fittest," only that form of wave motion, the trochoidal, best suited to indefinite propagation finally remains.

Naval Constructor D. W. Taylor, U. S. Navy, has suggested to the writer that the reason why a confused sea existing during a storm changes to a regular swell afterwards, may be due to more tangible causes than those just advanced.

For instance, in a heavy and rather confused sea, caused by the wind, the waves, generally speaking, are nearly parallel and traveling in the same direction. When the wind ceases entirely "we have throughout the area considered a lot of particles performing orbital motion, but with amplitude and velocity varying materially and orbit planes constantly changing. No external force being present, the viscosity of the water would tend to make these orbits identical, resulting in the regular trochoidal sea. Certainly if two adjacent particles are describing different orbits, the viscosity of the water would tend to make them describe the same, and there seems no reason why the same conclusion would not apply to the whole mass."

What precedes explains why waves which have traveled farthest, like swells from distant storms, are invariably the most regular in outline, and approach most closely the theo-

retical form. This fact will be clearly appreciated by comparing the photograph, page 216, which was taken in the midst of a very severe storm, with those on pages 62, 63, and 64, taken on the morning after a storm of about equal severity.

Each of the trochoidal subsurfaces through *b*, *c*, *d*, etc., fig. 1, page 16, is a surface of continuity, i. e., one which always passes through the same set of particles of liquid, so that between a pair of such surfaces is contained a layer of particles which are always the same.

These surfaces are also surfaces of uniform pressure. It can be shown that in a section made by a vertical plane perpendicular to the wave crest the area of the curvilinear

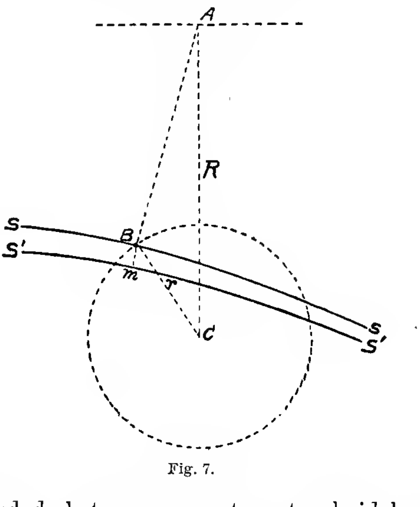


Fig. 7.

stratum included between any two trochoidal curves, as those through *a*, *b*, *c*, *d*, etc., fig. 1, page 16, and any two distorted verticals, as those through *k*, *l*, *m*, *n*, etc., is always equal to the area included between the corresponding bounding lines through *a*, *b*, *c*, *d*, etc., and *k*, *l*, *m*, *n*, etc., fig. 2, page 16, which inclose the same set of particles when the water is at rest.

Professor Rankine has shown mathematically that if through the orbit-center of any particle, as B, fig. 7, in a deep-water wave, the line AC be drawn in the direction of and proportional to gravity, using the same scale with which CB repre-

sents the direction and amount of the centrifugal force of the particle, the line AB will be normal to the surface of equal pressure through B and proportional to the resultant force acting upon the particle. This can only be true if AC represents the radius of the rolling circle of a trochoid through B.

The resultant of gravity and centrifugal force at any point B is represented by $g\,\frac{AB}{AC}$, and the excess of the uniform pressure for unit area at the subsurfaces S′, S′, infinitesimally near to SS, over that at SS, is $w\,\frac{AB}{AC}\,Bm$, w representing the weight of a cubic foot of water and Bm the perpendicular distance between the surfaces at the point B.

At the highest and lowest points of the trochoidal surfaces or subsurfaces the normal AB coincides in direction with AC, being represented for the surface SS by AC−CB at the crest and AC + CB at the hollow.

As the surfaces SS, S′ S′, indefinitely near together, are surfaces of uniform pressure, it follows that the normal thickness at any point B of the layer included between any two of them is inversely proportional to the resultant pressure AB.

At the mid height of the wave the vertical thickness of the layer is the same as that which it possessed in the undisturbed fluid.

Professor Rankine states the law governing the pressure upon individual particles as follows:

"The hydrostatic pressure at each individual particle during wave motion is the same as if the liquid were still."

The line AB measures the resultant force on the particle on the same scale as that by which AC represents gravity, and it is for wave motion what gravity is for still water, since the rate of increase of fluid pressure normal to the surface is measured at any point by the corresponding length AB.

It is for this reason called the *virtual gravity* at the point.

Virtual gravity has its minimum value at the crest and its maximum at the trough, and is equal to gravity at a point between the point of inflexion of the wave profile and the mid height for which the cosine of the angle ACB is $= \frac{r}{2R}$, i. e., when the triangle CAB is isosceles.

When a body floats freely upon the surface of a wave it

will be subjected to the same wave forces which would have been impressed upon the volume of water which it displaces, and if the floating body is a rigid one the effect produced upon it will be that due to the resultant of the forces which would have acted upon the particles composing the displaced volume of water had they remained in place. The resultant pressure of the water, therefore, against a body floating freely upon the surface of a wave is not equal to the weight W′ of the water displaced, acting vertically upward, but is changed both in direction and amount, so that it acts in a direction parallel to AB, with an intensity $W' \frac{AB}{AC}$.

In like manner, if a body is floating passively upon the surface of a wave, the reaction of the body against the water, instead of being only its weight, W′, acting vertically downward, is the resultant of gravity and centrifugal force, and becomes $W' \frac{AB}{AC}$, acting in a direction parallel to AB.

Sir William Henry White, in his Manual of Naval Architecture, states that Captain Mottez, of the French navy, reports that on long waves, about 26 feet in height, the apparent weight of a frigate at hollow and crest had the ratio of 3 to 2.

The rolling of a vessel at sea is caused by the continual change in direction of the resultant of gravity and centrifugal force, in its effort to place itself at any instant normal to the trochoidal subsurface corresponding to AB in the figure.

Theoretically the depth of a given particle in still water, below the level of the orbit-center of the same particle when wave motion is in progress, is—

$$\frac{\pi r^2}{L} = \frac{r^2}{2R}.$$

Actual measurements of the hydrostatic pressure at different depths below the crest of the wave are described in Chapter XI.

Early observers experienced great difficulty in obtaining, experimentally, the actual form of natural waves when of large dimensions, and their efforts to secure actual wave profiles were confined almost entirely to the small waves generated artificially in laboratories or canals. The methods employed for generating these waves and for securing the wave profiles have already been described in Chapter II.

Owing to the discovery of instantaneous photography, accurate photographs of waves can now be readily obtained, but in the case of deep-water waves of large size it is seldom possible to secure from a photograph an accurate section by a vertical plane perpendicular to the wave crest, owing to the fact that in deep water it is very difficult to get within the field of view a sufficient number of known heights and distances from which to determine accurately the wave profile. The writer has been unable to secure any observations or photographs showing accurately the actual cross section of a single deep-water wave of fair size.

Careful observations, however, indicate that these waves approach more closely to the theoretical wave form than do the corresponding shallow-water waves. This is only what would be expected, as waves of the former class are uninfluenced by the bottom, the character and propinquity of which are important factors in determining the form of the shallow-water wave.

Shallow-water waves.—As has already been explained in Chapter III, the orbits of the particles in shallow-water waves

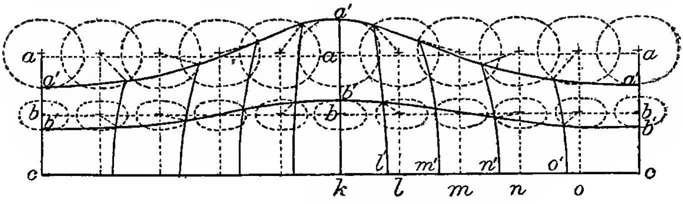


Fig. 8.

are ellipses, the eccentricity of which depend upon the ratio of the wave length to the depth of the water. The decrease of the semi axes of the orbits below the surface is given by equations (13), (14), and (15). The theoretical profile of a shallow-water wave at the surface, at mid depth and on the bottom, when $\frac{d_0}{L} = .2$, is shown in fig. 8.

The profile of the shallow-water wave coincides with that of the deep-water wave of equal height and wave length only at the highest and lowest points of the wave, the former profile lying wholly within the latter at other points, and the dif-

ference being most marked at mid height of the wave. As the ratio $\frac{d_0}{L}$ decreases, the theoretical difference in form between the two classes of waves becomes more marked.

When the bottom is sandy and the slope gradual and uniform, waves most frequently break about the time that $\frac{d_0}{L}=0.1$. At this time the relative theoretical dimensions of the orbits of the particles are as shown in the following table:

TABLE VII.—*Relative dimensions of orbits of particles for a wave whose length is ten times the distance from the line of centers of surface orbits to the bottom, the semi-minor axis of a surface particle,* $=\frac{h}{2}$, *being taken as unity.*

Distance of orbit centers below surface.	Relative dimensions of semi axes of elliptical orbits.		Distance of orbit centers below surface.	Relative dimensions of semi axes of elliptical orbits.	
	a'	b'		a'	b'
Surface	1.796	1.000	0.6	1.538	.380
0.1 of depth	1.736	.889	0.7	1.518	.283
0.2	1.684	.782	0.8	1.504	.187
0.3	1.638	.677	0.9	1.494	.094
0.4	1.599	.576	Bottom	1.491	.000
0.5	1.566	.477			

Theoretically the depth of a given particle in still water, below the level of the orbit center of the same particle when wave motion is in progress is $\frac{\pi a'b'}{L}$, a' and b' representing the semi axes of the elliptical orbit. Observations show that this theoretical value is almost invariably too small.

The difference in pressure between the crest and any other point of the shallow-water wave is—

$$\frac{a'^2-b'^2}{2a'R}\,b'\sin^2\theta.$$

From this equation it will be seen that the pressure at the crest and trough will be equal, but for intermediate points it will differ slightly, the greatest difference being at mid height of the wave.

Neither the conditions of continuity nor of dynamical equilibrium are exactly satisfied at all points of the theoretical shallow-water wave, being completely fulfilled only at the crest and trough.

FREE WAVE IN THE DULUTH CANAL, 1902.

FREE WAVE IN THE DULUTH CANAL, 1902.

Height, 10 feet, length, 210 feet.

The value for the molecular rotation of the shallow-water wave is—

$$\frac{\omega a' b'}{R^2 - a'^2},$$

an expression which is greater than that of the deep-water wave of the same height and length.

The surface profile of shallow-water waves can in many localities be obtained with ease and accuracy by means of instantaneous photographs taken while the wave is traveling parallel to the vertical face of a pier, breakwater, or other similar construction.

Such photographs were taken on several occasions in the Duluth Canal, and some of the results are shown on pages 62, 63, 64, and 65.

Accurate surface profiles have been constructed from these photographs, and are shown on Pl. II, the curve represented by the full line being in each case the profile constructed from the photograph, and that represented by the dotted line the theoretical surface profile for a wave of equal height and length traveling in water of the same depth, the direction of travel being as shown by the arrow at the top of the plate.

Fig. 1 shows a part of a wave 6 feet in height and 200 feet in length, followed by another 8.3 feet in height and 170 feet in length. (See photograph, p. 62.) The depth of the water is 25.9 feet, and 57 per cent of the wave height lies above still-water level and 43 per cent below this level. Theoretically, 55 per cent of the wave height should be above still-water level and 45 per cent below.

It will be noticed that the actual and theoretical profiles differ but little, except on the anterior slope of the higher wave.

Fig. 2 (see photograph, p. 63) shows a wave 10 feet in height and 210 feet in length, traveling in water 25.9 feet in depth, 61 per cent of the wave height being above still-water level and 39 per cent below. Theoretically, these quantities should be 56 per cent and 44 per cent, respectively. In this case the agreement between the actual and theoretical profiles is quite close, except in the vicinity of the mid height of the wave.

Fig. 3 (see photograph, p. 64) shows a wave 12 feet in height and 198 feet in length. The depth of the water is 26.1 feet; 66 per cent of the wave height is above still-water

level and 34 per cent below. Theoretically, these quantities should be 57 per cent and 43 per cent, respectively.

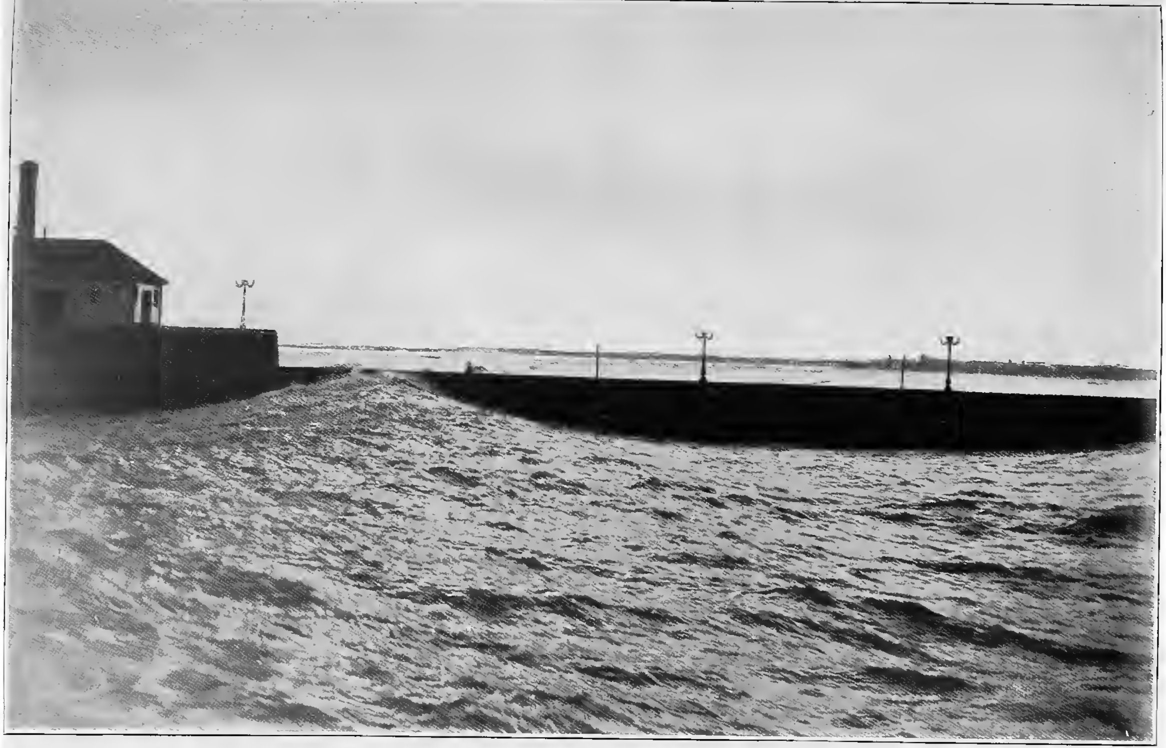
As is the case with all waves of considerable height in shallow water, the actual profile differs considerably from the theoretical profile, except at the highest and lowest points of the wave. The elevated portion of such a wave is always narrower and the depressed portion broader and flatter than is indicated by theory, and this difference becomes more marked as the wave approaches the point of breaking.

Fig. 4 (see photograph, p. 65) represents a wave 9.2 feet in height and 165 feet in length, in water 26 feet in depth. This wave is most unusual in one respect, i. e., 65 per cent of the wave height is *below* still-water level, whereas, theoretically, 56 per cent should be *above* still-water level. It therefore partakes somewhat of the character of a negative wave and is the only one out of several hundred observed exhibiting the peculiarity mentioned. Nothwithstanding this fact the actual profile agrees closely with the theoretical.

In figs. 1, 3, and 4 it will be noticed that the agreement between the actual and theoretical profiles is closer on the posterior slope of the wave than on the anterior slope. This is due to the fact that the anterior slope becomes steeper and more distorted than the posterior slope when the wave is traveling in shoal water, owing to friction on the bottom. This effect, although generally very noticeable in waves of such height, is for some unknown reason not shown in this particular photograph, which is therefore misleading to this extent.

It will be noticed that in every case the difference between the actual and theoretical profiles is greatest at mid height of the wave. This should be the case, for it is at this point that the shallow-water wave fulfills least satisfactorily the conditions of "continuity" and "dynamic equilibrium."

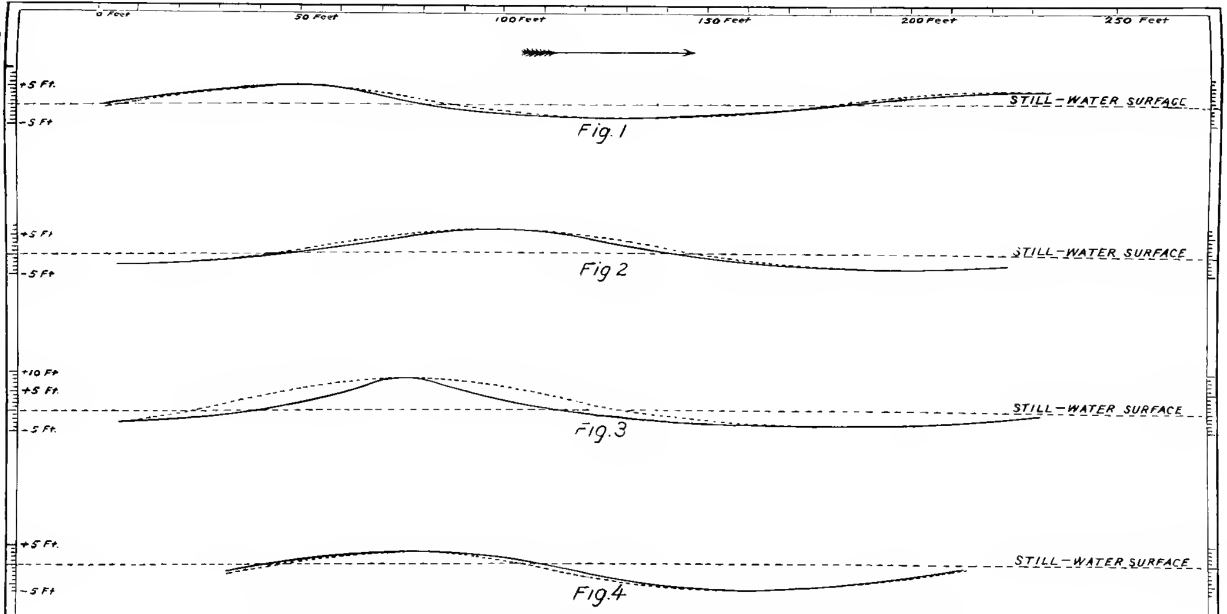
The photographs from which the profiles shown in figs. 1, 2, and 3 were constructed represent the typical waves of regular profile, which continue to run-in after severe storms, although the wind may have entirely subsided for some time previously. For example, the photograph, page 64 (see fig. 3), was taken about six hours after the wind had entirely subsided. About five hours previous to the time at which the photograph was taken, the largest waves were about 16 feet



FREE WAVE IN THE DULUTH CANAL, 1902.

Height, 12 feet; length, 198 feet.

PLATE II



SURFACE-PROFILES OF WAVES, DULUTH CANAL, MINN., 1901-2
Full lines represent profiles constructed from photographs.
Dotted lines represent theoretical profiles of waves of the same height and length.

WAVE IN THE DULUTH CANAL, 1901.

Height, 9.2 feet, length, 165 feet, wind velocity, 18 miles per hour, decreasing from 35 miles per hour.

FIG. 1.

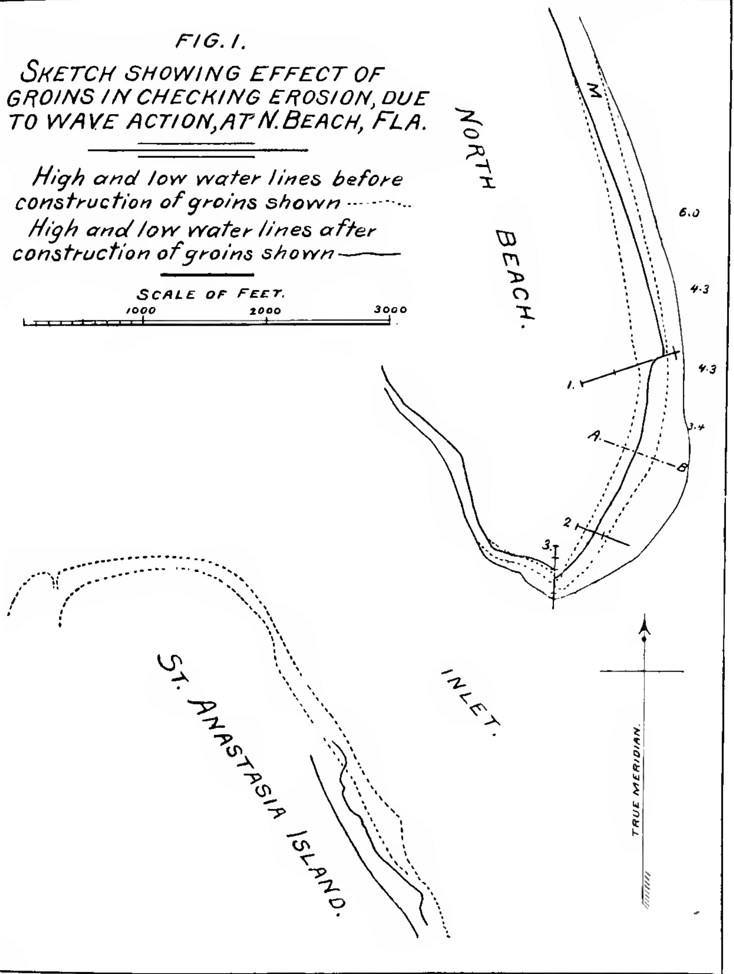
SKETCH SHOWING EFFECT OF GROINS IN CHECKING EROSION, DUE TO WAVE ACTION, AT N. BEACH, FLA.

High and low water lines before construction of groins shown ....

High and low water lines after construction of groins shown ————

SCALE OF FEET.

1000 2000 3000



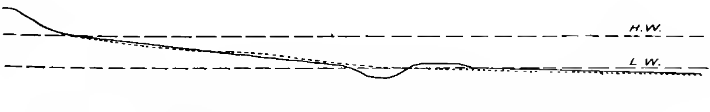


FIG. 2. SECTIONS ON A–B, FIG. 1, BEFORE CONSTRUCTION OF GROINS. Hor. scale, 1 in. = 125 ft. Ver. scale, 1 in. = 25 ft.
Dotted line shows condition in calm weather.
Solid " " " during storms.

ICE ON WEST BREAKWATER. UPPER ENTRANCE PORTAGE CANALS, MICH., APRIL 19, 1902.

in height, of about the same wave length as the one shown in fig. 3, and were almost as regular in profile. It was at about this phase of the storm that the only damage which was reported occurred.

Except for the abnormal relation of the position of the trough of the wave with respect to the still-water level, fig. 4 and the corresponding photograph represent a typical wave when the storm is decreasing in intensity. In this case the wind velocity had gradually decreased from 35 miles per hour, twenty hours previously, to 18 miles per hour at the time that the photograph was taken.

The irregularity of the wave form when a storm is at its height is shown in the photograph, p. 65, taken September 24, 1901, when the wind velocity was at its maximum, 46.5 miles per hour.

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CHAPTER V.

Height, Length, and Period of Waves.

Height depends on what? Height due to "fetch." Recorded heights of ocean waves. Corresponding lengths and periods. Similar data for waves on Great Lakes.

Causes tending to increase the height of waves. Maximum ratio of height to wave length. Reduction in height of waves under lee of a detached breakwater, or on passing into a close harbor.

The formation of waves being due generally to the action of the wind, their height at any point depends primarily upon the velocity and direction of the wind and the distance across open water from the windward shore, but is modified by the configuration of the adjacent shore lines, and by the depth of water in which the waves travel.

When a wave enters a bay, or indentation in the coast, of gradually decreasing depth, the height of the wave is sometimes reduced, and the length of crest considerably developed, the tendency of the wave being to become parallel to the interior shore line. When the wind blows in a direction parallel to the shore, the waves adjacent thereto are continually swinging around and traveling shoreward, instead of continuing in the direction in which the wind is blowing. If the shore line then changes direction, so as to extend nearly at right angles to the wind, the height of the waves will have been so reduced by dissipation against the parallel stretch of shore as to be less than would have been the case had the entire shore line been perpendicular to the direction of the wind, other conditions being supposed the same. The maximum distance across which the wind can act over open water of sufficient depth for wave formation is known as the "fetch," or line of maximum exposure.

Beginning at the windward shore, and assuming the wind velocity and direction as constant, the height of the waves must increase successively and fairly uniformly from zero to some limiting height, which the assumed wind velocity can just maintain.

When the "fetch" is great and the depth ample, the limit in wave height will generally be attained before the leeward shore is reached. The velocity of waves in deep water increases with the wave length, and as the waves increase in size out from the windward shore, their lengths increase correspondingly, and the wave velocity becomes greater. As the wind and waves are traveling in the same direction, the difference between their velocities (i. e., effective wind force) is continually decreasing. Finally, the waves attain such dimensions that their velocity can not be further increased by the wind, and, even if the latter subsides entirely, they are still capable of traveling as free waves long distances with undiminished velocity.

It is often noticed during very severe storms that the highest waves do not occur when the wind velocity is a maximum, but are seen soon after the wind begins to subside. This is explained by the fact that when the velocity of the wind is greatest, the tops of the waves are blown over, and the waves become very broken and irregular.

When the force of the wind diminishes somewhat, the broken and irregular waves tend to unite, forming a system of waves much larger and more regular than those previously existing.

Mr. Thomas Stevenson has established, from a large number of observations, an empirical formula for the maximum height of waves due to a given "fetch." This formula may be expressed as follows:

$$h=c\sqrt{f};$$

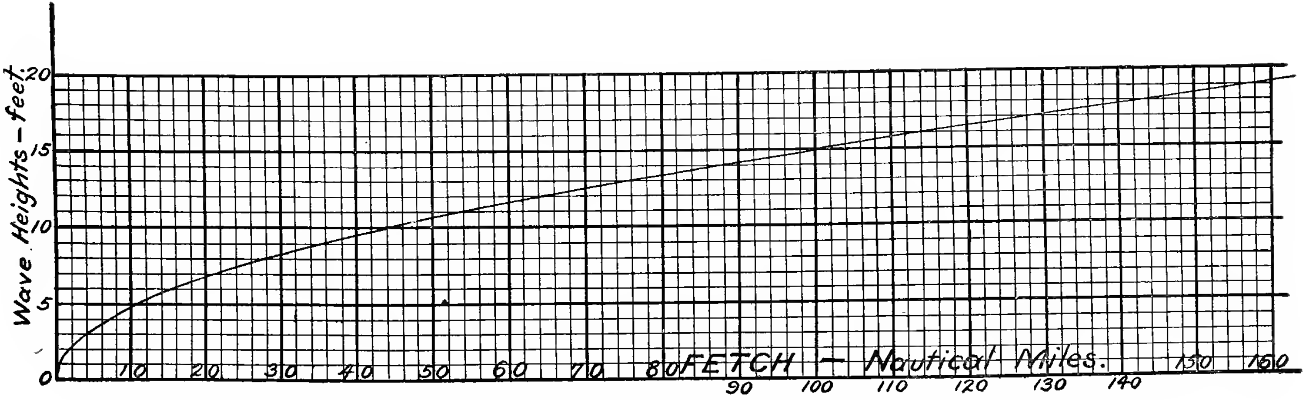
in which h=height of wave in feet; f=the "fetch," or distance to the windward shore, in nautical miles; and c=a coefficient, varying with the strength of the wind. That is, the height of the waves is proportional to the square root of their distance from the windward shore.

In the case of a strong gale, and when the water is of sufficient depth to allow the waves to be fully formed, the formula becomes—

$$h=1.5\sqrt{f}; \qquad (19),$$

which is the form in which it is generally used.

The curve represented by this equation is a parabola, and is shown graphically in fig 9.



CURVE SHOWING HEIGHT OF WAVES DUE TO "FETCH."

HEIGHT OF WAVE (FEET) = 1.5 $\sqrt{\text{FETCH (NAU. MILES)}}$ OR $h = 1.5\sqrt{f}$

Fig. 9.

Mr. Stevenson considered that for short reaches and violent squalls more exact results are given by the formula

$$h=1.5\sqrt{f}+(2.5-\sqrt[4]{f}); \qquad (19A),$$

h and f representing the same quantities as before.

It would seem that formula (19) has its limitations in the case of large oceans, where the "fetch" may be several thousand miles. It is not likely, as will be shown hereafter, that waves much higher than 45 feet often exist, and these, by the formula, correspond to a "fetch" of but 900 miles.

On the other hand, it may be doubted whether waves are often subjected to violent winds blowing in a single direction for a distance exceeding 900 miles, ocean gales having generally a rotary motion.

The following table gives the results of all available observations upon Lake Superior for height of waves due to "fetch."

TABLE VIII.—*Height of waves due to "fetch," arranged according to observed heights.*

Date.	Locality: Lake Superior and adjacent waters, and San Pedro Bay, Cal.	"Fetch" (nautical miles).	Height of wave.	
			Observed.	Computed.
1902.			*Feet.*	*Feet.*
Apr. 21	Duluth Basin	0.375	1.5	2.6
Apr. 21	.....do	.428	1.7	2.7
Apr. 21	.....do	.641	2.0	2.8
May 1	St. Louis Bay	.916	2.0	2.9
Apr. 22	Duluth Basin	.428	2.3	2.7
Apr. 22	Portage Lake	1.086	3.0	3.1
Apr. 22	Duluth Basin	.748	3.8	2.9
May 1	St. Louis Bay	1.923	4.5	3.4
	San Pedro Bay, Cal	15.64	6.0–7.0	5.9
Oct. 26	Stannard Rock	41.45	11.0	9.7
1895.	Marquette, Mich	116.63	[a] 15.0	16.2
1901.				
Sept. 16	Portage Breakwater	82.50	16.5	13.6
Sept. 24	Duluth Canal	258.62	23.0	24.1
	Total	....	92.8	92.6

[a] Estimated.

NOTE.—The first eight observations have been computed by equation (19A), and the last five by equation (19).

Considering the limited number of observations and the extended range in "fetch" and wave heights, the accordance between the observed and computed heights is satisfactory, especially since the formula employed in each case in comput-

ing the height was deduced originally from observations upon waves in salt water, which has a greater specific gravity than fresh water. It is to be regretted that there were no observations between "fetches" of 2 and 15.6 nautical miles, respectively, as these would have been of value in determining the limit of "fetch" to which equation (19A) is applicable.

Where the "fetch" and depth are sufficient, Admiral Coupvent Desbois has laid down a provisional theory based upon some 10,000 observations. He supposes that "the cube of the height of a wave is proportional to the square of the velocity of the wind."

In connection with what precedes it should be remembered that any deduction based upon the assumption that the wind is blowing for any considerable distance with the same velocity and in the same direction as at the observation station may be liable to considerable error, for both the velocity and the direction of the wind may be very different at the farther extremity of the reach considered. For example, during severe storms at Duluth, Minn., at the west end of Lake Superior, the velocity and direction of the wind at Houghton, Mich., on the south shore of the lake, and only 168 miles distant, are generally quite different,—in some cases little or no wind being reported at Houghton, when at Duluth the velocity varied from 30 to 40 miles per hour.

HEIGHT OF OCEAN WAVES.

Perhaps no characteristic of a wave impresses the beholder more than does unusual height, and it is this dimension which has been oftenest measured and described.

To the engineer a knowledge of the maximum wave height and length is of great importance, for, as shown in equation (12), the total energy of a wave varies nearly as the square of the height and as the first power of the length, and these dimensions for a given locality may be said to measure the capacity of the wave for destruction.

It may therefore be of interest to give briefly at this point some of the more important observations upon ocean waves.

Mr. Scott Russell gives 27 feet as the greatest height of wave observed by him in British seas. Prof. George B. Airy considers 30 to 40 feet to be the extreme height of unbroken waves.

In 1839 Commodore Wilkes, off Cape Horn, made accurate observations upon a very regular series of waves, which he found to have a height of 32 feet, a length of 380 feet, a velocity of 43.6 feet per second, and period of 8.7 seconds.

It will be noticed by comparison with other observations that these waves were unusually steep for waves of such length.

In the report of the British Association for 1850 Dr. William Scoresby gives the result of wave measurements carefully made by him on various occasions. He states that on the Atlantic Ocean on March 4, 1848, at least half of the waves passing the ship were far above the level of his eye (30.25 feet above the water), and long ranges of waves were frequently observed extending about 100 yards on one or both sides of the vessel (the sea coming right aft) and rising about $2\frac{1}{2}^\circ$ above the visible horizon when the wave summit was about 100 yards, or nearly 13 feet higher than the eye of the observer, making the total height of such waves about 43 feet. This remarkable height was attained by at least one out of every six waves, while peaks of crossing waves and crests of breaking waves would shoot up 10 or 15 feet higher. Doctor Scoresby therefore concluded that the crests of the mean highest waves, not including pointed or broken waves, were about 43 feet above the level of the hollow occupied by the ship. The greatest distance from crest to crest observed by Doctor Scoresby was 790 feet, the mean distance 559 feet, and the interval of time between waves sixteen seconds.

Mr. Vernon Harcourt states that the extreme height of waves in the Atlantic is about 40 feet.

A large number of observations, embracing the height, length, and period of ocean waves, have been taken by officers of the United States Navy. Through the courtesy of the United States Hydrographic Office the writer has been permitted access to many of these observations, and has incorporated a number of them in Table IX.

In an article read before the Physical Society, February, 1888, Hon. Ralph Abercromby described wave observations taken by him on board the steamship *Tongariro* in various parts of the South Pacific between New Zealand and Cape Horn, in 1885. In latitude 51° S., longitude 160° W., waves from 21 to 26 feet in height were measured. The lengths and veloci-

ties of waves of the same system, measured on deck just before the heights were determined, varied in the case of the former from 358 to 507 feet, and in the case of the latter from 28.5 miles to 32 miles per hour. The sea was classed as 6 or 7 on the ordinary scale of 0 to 8, and is stated to have been a fair average sea in the South Pacific. In July, 1885, in latitude 55° S., longitude 105° W., waves were measured varying in height from 28.5 to 46 feet. The lengths and velocities of waves of the same system, measured as in the previous case, varied from 445 to 765 feet for the lengths and 35.5 to 47.5 miles per hour for the velocities.

Abercromby states that "this was the heaviest sea encountered during the whole voyage, but did not appear excessively so to the eye. Under these circumstances the height of 46 feet given by one set of observations seems excessive. Any error in the true height must come from the estimate of 6 feet for the difference of the height of the eye on the crest and in the trough." It was estimated from observations and measurements that the water in the trough was 6 feet farther below the eye than that in the crest. It was noticed that the relation of length and velocity to height was very irregular, but this was due to the character of the waves and not to errors of observation. The heights were determined by a very delicate aneroid barometer, and the author states that its errors probably never exceeded 2 to 2.5 feet, while those of estimation as to height of water in the trough and on the crest, below the eye, might be 2 feet either way. He states that he has seen a heavier sea in the Atlantic, and that if only 40 feet be taken as the highest wave of the series observed it is, in his opinion, perfectly certain that much greater heights are sometimes attained, and he is convinced that 60 feet at least from trough to crest must be attained by exceptional waves. He expresses the opinion that wave lengths of 1,902 and 2,703 feet, which have been given by two observers, must have resulted from the interference of following waves.

Through the kindness of Commander Z. L. Tanner, U. S. Navy, the writer has been furnished with a remarkable photograph of a wave, taken on board the United States Fish Commission's steamer *Albatross*, when in the Pacific Ocean, off the west coast of North America, in a depth of from 200 to 300 fathoms. On this photograph the foreyard of the *Alba-*

tross is shown projected parallel to the crest of a huge wave and a little below it. The length of the *Albatross* on the water line is 200 feet, the distance of the foremast from the bow is 68 feet, height of foreyard above still water 45 feet, distance of camera in rear of foremast 142 feet, and height of camera above still water 20 feet. The estimated distance of the wave crest is not given. The photograph shows that the crest of the wave shuts off the horizon, but whether the camera was much below the level of the crest or was in the same horizontal plane with it can not be determined. Assuming that the latter condition existed, and also that the bow was at the lowest point of the preceding wave hollow—neither of which assumptions is probable, but is made in order to determine the minimum wave capable of producing the photographed effect—it is found that the crest of this minimum wave is 57 feet above the (still) water line of the vessel at the bow. Even if we suppose the bow to be submerged several feet below its normal position, it seems reasonably certain that the photographed wave could not have been less than 50 feet in height, and was probably much higher. Captain Tanner states that in a northwest gale off Cape Horn waves were seen of an estimated height of a little more than 30 feet and of a length between 700 and 800 feet.

Dr. Gerhard Schott, during a trip from Bremen to Japan and return, 1891–92, made wave observations, the results of which were given in an interesting article, entitled "Uber die Dimensionen der Meereswellen. Nach eigenen Beobachtungen von Dr. Gerhard Schott. Berlin, 1893."

For measuring the height of waves Doctor Schott used a very sensitive aneroid with microscopic reading. The greatest height of storm waves recorded by him was 12 meters, or a little over 39 feet. The highest waves recorded during the more extensive observations of Lieutenant Paris rose 11.5 meters (37.7 feet) above sea level. Doctor Schott is of the opinion that observations of waves, in the stormiest weather and in the open sea, that are above 15 meters (49.2 feet) in height are extremely rare. He thinks that the reports of mariners or travelers of waves 60 or 70 feet high are almost invariably erroneous. With an average good breeze the distance between waves, according to Dr. Schott's observations, was about 115 to 130 feet, and the time between waves about

4.5 to 5 seconds. That is, about every 5 seconds a new wave would come, traveling at the rate of about 22 to 26 feet in a second. The heaviest storm waves he met moved about 60 feet in a second. They attained a length of about 700 feet. According to Doctor Schott, the rapidity of wave movement does not increase proportionally with an increase in the strength of the wind. He states that storm waves have a steeper front than average waves, and confirms the determinations of Lieutenant Paris, who found that the proportion between wave heights and wave lengths in a high sea is as 1 to 18, and in a moderate sea as 1 to 33.

In January, 1894, the hurricane deck of the steamship *Normania* was swept by a huge wave, which wrecked the officers' rooms and three large saloons, and carried away 14 ventilators, rendering it necessary for the vessel to put back to New York, although about one-third of the way across the Atlantic when the damage occurred. From the known height of the deck above water it was determined that this wave was at least 40 feet in height.

In 1895 Lieut. Oskar Gassenmayr, of H. I. M. S. *Donaŭ*, made a number of observations upon the height, length, and period of waves in the South Atlantic.

Dr. Vaughan Cornish, in a paper read before the Royal Geographical Society of Great Britain in 1901, entitled "Travels in search of waves in 1900," states that in the North Atlantic during a storm he measured some waves 40 feet in height. These were exceptional, but waves of over 30 feet in height were common, though the average height was not more than 18 feet. He calls attention to the fact that the published records usually give only the average height, which rarely exceeds 28 feet in the heaviest gales. This average, however, he considers consistent with the occurrence every few minutes of waves 45 to 55 feet high, which are often reported by sailors and discredited by landsmen.

In the Southern Indian Ocean, between the Cape of Good Hope and the island of St. Paul, thirty waves have been measured averaging 29.5 feet in height, the largest being 37.5 feet high. Six waves of this height followed one another with remarkable regularity.

East of the Cape of Good Hope, during strong west winds

which blew with great regularity for four days, the height of the waves only increased from 19.7 to 23 feet.

Ocean waves in shallow water.—The height of the largest waves which have at various times assailed and damaged Wick Breakwater was estimated by the resident engineers as 42 feet from hollow to crest.

The largest waves at Peterhead, North Britain, during the severe storm of February 15–16, 1900, described in detail in Chapter VI, were probably 35 feet in height and about 600 feet in length, with a period of 15 seconds.

At the same locality, in November, 1888, Mr. William Shield measured waves 26 feet in height and 500 feet in length, with periods of 12.2 seconds, the depth of the water being about 45 feet.

At Algoa Bay, in water 24 feet in depth, he measured waves 10 feet in height and 200 feet in length, with periods of 10 seconds.

For the convenience of those interested in the subject, the following table has been prepared, showing the results of some of the more important observations upon ocean waves.

TABLE IX.—*Showing height, length, and period of observed ocean waves, arranged according to height.*

Date.	Locality.	Force of wind.	State of sea.	Wave. Height.	Wave. Length.	Wave. Period.	Ratio of length to height $=\frac{L}{h}$	Authority.
				Feet.	*Feet.*	*Seconds.*		
	North Pacific			(*a*)				Photograph from Capt. Z. L. Tanner, U. S. Navy.
1885	South Pacific		Heavy	46.0	765	16.5	16.6	Abercromby.
1848	Atlantic			43.0	559	11.7	13.0	Wm. Scoresby.
1894	do			40.0				Officers *Normania*.
	Wick Bay			40.0				Thos. Stevenson.
1900	North Atlantic			40.0				Vaughan Cornish.
1891	South Atlantic	10.0	Confused	39.4	701	11.7	17.8	Doctor Schott.
1900				37.7				Lieutenant Paris.
1900	Peterhead	*b*50–89		35.0	600	15.0	16.0	Wm. Shield.
				36.0				*Novara.*
	South Pacific			33–36				Captain Chüden.
	Indian Ocean			33.6	374	7.5	11.1	Lieutenant Paris.
1891	do	9.0	Confused	32.8	424	9.1	12.9	Doctor Schott.
1839	South Pacific			32.0	380	8.7	11.9	Commodore Wilkes.
1891	South Atlantic	9.0	Confused	29.5	370	8.5	12.5	Doctor Schott.
				27.0				Scott Russell.
1892	Indian Ocean	9.0	Confused	26.2	428	8.8	16.3	Doctor Schott.
1888	Peterhead			26.0	500	12.2	19.2	Wm. Shield.
1886	Atlantic	10.0	Confused	25.0	375	7.5	15.0	Officers U. S. Navy.
1885	Indian Ocean	4.0	do	25.0	450	11.0	18.0	Do.
1885	do	4.0	do	25.0	500	13.0	20.0	Do.

1892	do	5.0	Regular	24.6	1,121	14.5	45.6	Doctor Schott.
1895	South Atlantic	7.5		24.6	459	11.0	18.7	Lieutenant Gassenmayr.
1885	Indian Ocean	4.0	Confused	24.0	400	12.0	16.7	Officers U. S. Navy.
1886	Pacific	2.0	Long swell	23.0	350	10.0	15.2	Do.
1885	Indian Ocean	4.0	Confused	22.0	400	10.0	18.2	Do.
1895	Atlantic	6.5		21.3	394	10.0	13.8	Lieutenant Gassenmayr.
1886	do	8.0	Regular	21.0	328	8.0	15.6	Officers U. S. Navy.
1885	Indian Ocean	4.0	Confused	21.0	400	9.0	19.0	Do.
1892	do	5.0	Regular	19.7	460	9.5	23.4	Doctor Schott.
1895	Atlantic	9.0		19,7	262	9.5	13.3	Lieutenant Gassenmayr.
1886	do	7.0	Regular	18.0	318	7.5	17.7	Officers U. S. Navy.
1885	Indian Ocean	4.0	Confused	18.0	300	8.0	16.7	Do.
1885	do	4.0	do	18.0	250	9.0	13.9	Do.
1885	do	4.0	do	18.0	400	12.0	22.2	Do.
1891	do	5.0	Regular	16.4	396	8.8	24.1	Doctor Schott.
1885	do	4.0	Confused	16.0	350	9.0	21.9	Officers U. S. Navy.
1885	do	1.0	do	16.0	350	11.0	21.9	Do.
1885	do	4.0	do	15.0	200	8.0	13.3	Do.
1885	do	4.0	do	15.0	150	7.0	10.0	Do.
1885	do	4.0	do	15.0	300	9.0	20.0	Do.
1885	do	4.0	do	15.0	250	9.0	16.7	Do.
1895	South Atlantic	7.0		14.8	394	8.0	26.6	Lieutenant Gassenmayr.
1895	Atlantic	8.0		14.8	328	8.0	22.2	Do.
1892	South Atlantic	6.0	Confused	14.8	202	6.0	13.6	Doctor Schott.
1895	do	6.0		14.4	459	10.0	31.9	Lieutenant Gassenmayr.
1883	China Sea	6.0	Confused	14.0	160	6.0	11.4	Officers U. S. Navy.
1892	South Atlantic	0.0	Regular	13.1	571	10.0	43.6	Doctor Schott.
1890	do	6.0	Confused	13.1	193	6.6	14.7	Do.
1886	Pacific	4.0	do	12.0	276	5.0	23.0	Officers U. S. Navy.

a Greater than 50.

b Miles per hour.

TABLE IX.—*Showing height, length, and period of observed ocean waves, arranged according to height*—Continued.

Date.	Locality.	Force of wind.	State of sea.	Wave. Height.	Wave. Length.	Wave. Period.	Ratio of length to height $=\frac{L}{h}$	Authority.
				Feet.	*Feet.*	*Seconds.*		
1895	South Atlantic	3.0		11.5	426	10.0	37.0	Lieutenant Gassenmayr.
1896	do	5.0		11.5	164	6.0	14.3	Do.
1895	Atlantic	8.0		11.5	148	7.0	12.9	Do.
1895	South Atlantic	4.0		11.1	180	5.5	16.2	Do.
1883	China Sea	6.0	Confused	11.0	167	6.8	15.2	Officers U. S. Navy.
1883	Pacific	8.0	Irregular	10.5	209	6.7	19.9	Do.
	Algoa Bay	a 58.0		10.0	200	10.0	20.0	Wm. Shield.
1895	South Atlantic	5.0		9.8	426	9.5	43.5	Lieutenant Gassenmayr.
1895	do	6.0		9.8	148	6.6	15.1	Do.
1895	Atlantic	6.0		8.9	115	6.0	12.9	Do.
1891	Indian Ocean	0.0	Regular	8.2	305	7.0	37.2	Doctor Schott.
1895	South Atlantic	6.0		8.2	131	6.5	16.0	Lieutenant Gassenmayr.
1892	Indian Ocean	6.0	Confused	8.2	145	6.4	17.7	Doctor Schott.
1895	South Atlantic	5.0		8.2	262	6.0	32.0	Lieutenant Gassenmayr.
1895	do	6.0		8.2	131	5.5	16.0	Do.
	Bishop Rock			8.0	171	7.5	21.4	Sir Jas. N. Douglass.
1883	Pacific	6.0	Confused	8.0	191	6.1	23.9	Officers U. S. Navy.
1886	do	1.0	Swell	8.0	400	10.0	50.0	Do.
1883	South Pacific	5.0	Confused	8.0	249	7.0	31.1	Do.
1885	Pacific	4.0	do	8.0	100	7.0	12.5	Do.
1895	South Atlantic	6.0		7.5	131	6.0	17.5	Lieutenant Gassenmayr.
1895	do	6.0		7.5	115	6.0	15.3	Do.

1885	South Pacific	2.0	Regular	7.0	100	6.0	14.3	Officers U. S. Navy.
1892	North Atlantic	0.0	do	6.6	328	7.4	49.7	Doctor Schott.
1895	South Atlantic	6.0		6.6	98	6.0	14.8	Lieutenant Gassenmayr.
1892	do	5.0	Confused	6.6	123	5.0	18.6	Doctor Schott.
1892	do	2.0		6.6	158	6.3	23.9	Lieutenant Gassenmayr.
1885	South Pacific	3.0	Regular	6.0	100	6.0	16.7	Officers U. S. Navy.
1885	Pacific	1.0	do	6.0	80	5.0	13.3	Do.
1886	Atlantic	4.0	do	5.0	261	7.7	52.2	Do.
1886	do	0.1	do	5.0	314	8.3	62.8	Do.
1885	Pacific	1.0	do	5.0	60	5.0	12.0	Do.
1895	South Atlantic	4.0		4.9	131	5.0	26.7	Lieutenant Gassenmayr.
1895	do	5.0		4.9	98	4.5	20.0	Do.
1895	do	5.0		4.6	115	4.8	25.0	Do.
1895	do	4.0		4.6	148	5.0	32.2	Do.
1895	do	4.0		4.3	82	4.0	19.1	Do.
				4.0	500	10.0	125.0	Deverell.
1887	Caribbean	3.0	Regular	4.0	100	8.0	25.0	Officers U. S. Navy.
1885	Pacific	0.0	do	4.0	50	5.0	12.5	Do.
1892	Indian Ocean	2.0	do	3.9	134	5.0	34.4	Doctor Schott.
1895	South Atlantic	6.0		3.6	115	5.2	31.9	Lieutenant Gassenmayr.
1892	do	5.0	Confused	3.3	119	4.9	36.1	Doctor Schott.
1887	Caribbean	3.0	Regular	3.0	50	4.0	16.7	Officers U. S. Navy.
1887	Atlantic	4.0	do	3.0	30	3.0	10.0	Do.
1892	North Atlantic	1.0	do	2.6	162	5.2	62.3	Doctor Schott.
1892	Indian Ocean	5.0	Confused	2.6	108	4.6	41.5	Do.
1887	Caribbean	3.0	Regular	2.0	40	4.0	20.0	Officers U. S. Navy.

[a] Miles per hour.

HEIGHT, LENGTH, AND PERIOD OF WAVES UPON THE GREAT LAKES.

The writer has been unable to find any published data relating to the length or period of waves upon the Great Lakes, and but little relating to their height. Practically all available published information upon the subject is embraced in what follows:

Col. T. J. Cram, U. S. Corps of Engineers, stated that the greatest waves at Buffalo, N. Y., on Lake Erie, were 10 feet in height from hollow to crest. (See Report of the Chief of Engineers, U. S. Army, for 1868, p. 228.)

At Eagle Harbor, Lake Superior, in 1875, Mr. L. Y. Schermerhorn, U. S. assistant engineer, measured waves 10 to 12 feet in height from hollow to crest. (See Report of the Chief of Engineers, U. S. Army, for 1876, p. 329.)

In the Report of the Chief of Engineers, United States Army, for 1885, pages 2279–2280, Lieut. Col. H. M. Robert, U. S. Corps of Engineers, states that in 1884, at Oswego Harbor, Lake Ontario, an effort was made to obtain data relating to the height, velocity, and force of waves thrown upon the western part of the west breakwater.

The height of the waves was determined by a level placed upon the shore and directed lakeward upon the incoming waves.

The velocities were determined by the time required for the waves to sweep along the shorearm of the breakwater.

The results of the observations are summarized as follows: "During a severe gale from the northwest the waves attained a height of from 14 to 18 feet above the normal surface of the lake, with a velocity of from 30 to 40 miles per hour" (44 to 58.7 feet per second).

The "height of the waves" as here used, evidently refers to the level of the crest above "the normal surface of the lake," and not to the wave height from hollow to crest. As the level of the lake surface during severe storms usually varies considerably from the normal, it is not possible from the data given to arrive at the actual height of these waves from hollow to crest.

The velocities given are considerably in excess of any yet measured upon Lake Superior.

Maj. Charles E. L. B. Davis, U. S. Corps of Engineers, takes 12 feet as the maximum height of waves at the harbor of

refuge, Milwaukee, Wis., on Lake Michigan. (See Report of the Chief of Engineers, U. S. Army, for 1891, p. 2557.)

At the same place (see Report of the Chief of Engineers, U. S. Army, for 1894, p. 2088) Lieut. C. H. McKinstry, U. S. Corps of Engineers, states that the maximum height of waves is "about 13 feet."

In but two of the preceding cases is it specifically stated that the results given are based upon actual measurements.

In 1901 and 1902 the writer instituted a series of wave measurements in the Duluth Ship Canal and in the adjacent waters of Lake Superior.

In all, 650 measurements of wave length, height, and period were made, on 28 different dates, upon waves varying in height from 2 feet to 23 feet, in length from 45 feet to 275 feet, and in period from 3.8 seconds to 9.6 seconds. The corresponding still-water depths ranged from 3.3 feet to 26.7 feet.

The results of these measurements are given in the following table:

TABLE X.—*Height, length, and period of waves observed in the Duluth Ship Canal and in Lake Superior, arranged according to mean depth of water.*

OBSERVATIONS TAKEN IN DULUTH SHIP CANAL.

Date.	Heights.	Lengths.	Periods.	Mean depth.	Ratio of length to height $= \frac{L}{h}$	Number of observations.
1901.	*Feet.*	*Feet.*	*Seconds.*	*Feet.*		
May 23	8.0-17.0	160-200	6.9-8.0	24.7-26.7	20.0-10.6	39
May 24	6.5- 8.0	130-150	6.0-7.5	24.7-26.7	20.0-18.8	3
June 14	9.0-12.0	130	5.1-5.8	24.7-26.7	14.4-10.8	20
June 15	2.0- 8.0	120-140	5.9-6.4	24.7-26.7	60.0-17.5	21
July 24	4.0-12.0	110-150	5.8-6.8	24.7-26.7	27.5-12.5	40
August 9	4.0-14.0	100-130	4.4-6.8	24.7-26.7	27.5- 9.3	49
September 24	6.0-23.0	200-250	7.4-8.0	24.7-26.7	33.3- 9.1	80
October 9	5.0-10.0	140-150	6.3-6.5	24.7-26.7	28.0-15.0	30
October 10	4.0- 9.0	150	6.8-8.3	24.7-26.7	30.0-16.7	20
November 6	6.0-13.0	130-150	4.9-5.4	24.7-26.7	21.7-11.5	20
November 22	7.0-16.0	150	5.5-6.4	24.7-26.7	21.4- 9.4	21
1902.						
April 21	7.0-14.0	100-200	5.1-8.7	24.7-26.7	25.0-10.0	75
April 22	7.0-13.5	175-275	6.0-9.1	24.7-26.7	30.6-13.8	22
May 1	6.5-11.7	100-150	4.4-6.0	24.7-26.7	28.0-10.5	15
May 20	8.0-14.5	110-200	5.5-8.0	24.7-26.7	25.0-10.0	13
August 15	2.5	70	3.9	24.7-26.7	28.0	3
October 23	7.0-13.0	130-200	5.0-8.0	24.7-26.7	24.3-12.0	14
October 25	9.0-13.0	105-185	5.0-7.0	24.7-26.7	20.6-10.0	14
October 26	8.0-10.0	170-200	6.2-6.8	24.7-26.7	21.7-19.0	3
November 12	12.0-16.0	160-250	6.1-9.2	24.7-26.7	20.8-14.3	23
November 13	10.0-14.0	188-200	7.5-8.0	24.7-26.7	15.7-14.3	5

OBSERVATIONS TAKEN IN LAKE SUPERIOR.

1902.						
April 22	7.0–12.0	160–185	7.4–8.6	18.7	25.7–14.2	7
21	7.0– 9.7	150–184	6.9–8.0	15.7–15.9	21.4–17.2	4
May 20	4.0–13.0	140–187	7.5–9.0	15.7–15.9	40.0–13.3	26
April 22	4.0–10.0	140–180	6.7–8.7	13.0–14.0	30.0–16.0	14
September 13	7.5	120	6.0	13.0–14.0	16.0	3
October 23	5.0–10.0	89–164	5.5–9.6	13.0–14.0	30.0– 9.4	15
25	6.0–10.0	94–139	4.5–7.1	13.0–14.0	19.9– 9.9	9
June 5	2.5– 4.0	68– 73	3.9–6.5	6.9– 7.0	30.0–17.5	9
April 26	3.5– 4.5	75– 90	5.0–5.6	6.3	20.4–20.0	5
23	3.0	75	5.8	5.1– 5.7	25.0	3
June 4	3.0	50	4.0	5.1– 5.7	16.7	2
5	2.5– 3.5	50– 70	3.8–5.6	5.1– 5.7	30.0–16.7	10
April 23	2.5– 2.75	60– 70	5.4–6.2	4.0– 4.2	25.5–24.0	4
May 31	2.5	60	4.8	4.0– 4.2	24.0	2
Do	2.0	45	4.9	3.3	22.5	4
June 3	2.5	50	4.8	3.3	20.0	3
Total						650

On September 24, 1901, waves were measured at the outer entrance of the Duluth Canal of a height of 23 feet from hollow to crest, but at the time that these particular measurements were in progress there was an opposing current of from 2 to 3 feet per second, which undoubtedly increased the height and decreased the length of the waves, but to what extent is not known.

All of the observations upon storm waves in 1901–2 were necessarily made upon waves in relatively shallow water, and no observations are available for similar waves in the deeper waters of Lake Superior. As the result of inquiries of vessel captains who have navigated Lake Superior for many years, and who have, in some cases, made special note of the fact that in unusually severe storms the horizon could not be seen from the wheelhouse when the ship was in the trough of the sea, on account of adjacent wave crests, it seems probable that during unusually severe storms upon Lake Superior, which occur only at intervals of several years, waves may be encountered in deep water of a height of from 20 to 25 feet and a length of 275 to 325 feet.

Owing to its greater size and depth, larger waves are found upon Lake Superior than upon any other of the Great Lakes.

CAUSES TENDING TO INCREASE THE HEIGHT OF WAVES.

When a wave encounters an opposing current its height is increased, its length decreased, and its velocity consequently diminished. The wave front becomes steeper and steeper, and the wave frequently breaks, either in part or as a whole, presenting much the same appearance as if it broke on a bank or shoal.

Observations taken in the outer end of the Duluth Canal August 9, 1901, when there was no current, showed that the maximum waves had a height of 12 feet. Similar observations at the same locality, taken half an hour later, when the velocity of the current opposed to the waves was about 2 feet per second, showed that the maximum waves were 14 feet high. The wind conditions were practically the same in both cases.

Scott Russell found, from experiment, that the height of a positive wave might be increased indefinitely by propagation in a channel of uniform depth but gradually decreasing width the height of the wave varying nearly inversely as the square root of the breadth of the channel.

Prof. G. B. Airy has shown mathematically that in a channel of uniform breadth but of gradually diminishing depth, the height of the wave varies inversely as the fourth root of the depth.

Bazin, in 1859, instituted a series of experiments upon positive waves propagated in a regular channel 6½ feet in width, with a bottom sloping uniformly at the rate of about 1½ feet in 1,000 feet.

Stations were established at distances of 60 to 65 feet apart, and the height and velocity of the wave were noted.

The results of two sets of observations, taken as just described, are given in the following table. The fourth and fifth columns have been computed by the writer, the fourth according to Professor Airy's theoretical deduction and the fifth according to the inverse squares of the depth, the height and depth corresponding to the seventh point of observation being used in each case as the base of the computations:

Observations by Bazin, 1859.

Number of points of observation.	Depth of water before passage of wave.	Observed height of wave.	Computed height of wave. $h \propto \frac{1}{\sqrt[4]{d}}$	Computed height of wave. $h \propto \frac{1}{\sqrt{d}}$	Velocity of wave.
	Feet.	*Feet.*	*Feet.*	*Feet.*	*Feet per second.*
1	2.24	0.39	0.33	0.31	....
2	2.16	.30	.34	.31	8.70
3	2.04	.30	.34	.32	8.88
4	1.95	.30	.35	.33	8.26
5	1.85	.33	.35	.34	8.67
6	1.75	.36	.35	.35	8.68
7	1.64	.36	.36	.36	[a] 8.32
8	1.55	.36	.37	.37	8.12
9	1.46	.39	.37	.38	7.80
10	1.35	.39	.38	.40	7.85
11	1.45	.39	.37	.38	7.54
12	1.12	.43	.40	.43	7.46
13	0.80	.52	.43	.52	6.69
Total	....	4.82	4.74	4.80	....
1	2.08	.39	.57	.52	....
2	2.00	.46	.58	.53	8.81
3	1.88	.52	.59	.55	8.64
4	1.79	.62	.59	.56	9.43
5	1.69	.62	.60	.58	[b] 8.04
6	1.59	.62	.61	.60	7.62
7	1.48	.62	.62	.62	9.00
8	1.39	.62	.63	.64	7.65
9	1.30	.62	.64	.66	7.99
10	1.19	.62	.66	.69	7.57
11	1.29	.56	.64	.66	7.46
12	.96	.62	.69	.77	7.46
13	.64	.36	....	....	[c] 7.13
Total	....	[d] 6.89	7.42	7.38	....

[a] The wave did not break until a little past the thirteenth point of observation.
[b] The wave broke between the twelfth and thirteenth points of observation.
[c] Wave broken.
[d] The last observation omitted in getting total. The wave traveled about 100 feet before passing the first point of observation.

It will be seen from the preceding table that in the particular observations described the change in heights in each of the two cases varied more nearly with the inverse square roots of the depths than with the inverse fourth roots, as they should, according to Professor Airy's deductions. But it is quite possible that the waves in Bazin's channel of but $6\frac{1}{2}$ feet in width were higher than they would otherwise have been, on

account of the tendency of waves running along a vertical wall to rise up against it to a greater height than would be attained in open water, as explained hereafter.

The effect of decreasing breadth and diminishing depth upon the height of waves is beautifully illustrated in bays, gulfs, and channels like the Bay of Fundy, the Gulf of California, and Bristol Channel, where the tidal rise near the head is several times greater than that at the mouth.

When unbroken waves come in contact with a vertical wall of sufficient height above the water surface, they are reflected by it, and the action of the particles is stated by Rankine to be as follows:

The particles in contact with the wall move up and down through a height equal to twice the original height of the waves, and so do those at half a wave length from the wall. The particles at a quarter of a wave length from the wall move backward and forward horizontally, and intermediate particles oscillate in lines inclined at various angles. (See also Ch. III.)

Waves are reflected by surfaces inclined as much as 45° to the horizontal, provided the height of the structure is sufficiently great to prevent an appreciable part of the wave from passing over it.

Somewhat analogous to the preceding case is the action of a wave running along in a direction parallel to a vertical wall, or one not greatly inclined. Owing to friction against the wall, and to the usual decrease of depth near the wall, the height of the wave at the wall is increased very markedly, but it falls off to its normal condition at a short distance from the wall.

Near the outer end of the Duluth Canal, Minnesota, during ordinary storms waves from 8 to 12 feet in height and running parallel to the piers have their heights alongside the piers increased from 2 to 3 feet by the action just described.

This phase of wave action is of great importance in connection with the preparation of plans for parapet walls of breakwaters, piers, docks, etc.

In the case of oscillatory waves in a canal of triangular cross section, whose sides are inclined at an angle of 45°, a section made by a vertical plane perpendicular to the axis of the canal is the common catenary, with its vertex in the plane of the middle of the canal and its concavity turned upward

or downward, according as the section is taken where the water is elevated or depressed, i. e., in the vicinity of the crest or hollow. For example, the wave ridge lies in a vertical plane and is higher toward the slanting sides of the canal than in the middle.

Professor Kelland shows that the velocity of a wave in such a canal is given by equations (13) and (16), provided d_0 is equal to half the greatest depth in the triangular cross section, and that for L is substituted a quantity less than the length of the waves in the canal in the ratio of $1 : \sqrt{2}$.

Theoretically, the maximum possible height of a wave in open water depends upon the centrifugal force of the surface particles, or $\frac{4\pi^2 r_s}{t^2} = \frac{2\pi v'}{t}$. It may be shown that $R : r_s :: g : \frac{4\pi^2 r_s}{t^2}$, from which expression it is seen that when the centrifugal force of a particle becomes equal to gravity, $r_s = R$. At this instant the resultant force of a particle at the highest point of the wave is zero, and for any further increase of the centrifugal force, the wave length remaining the same, the particle would fly from its orbit and the wave would commence to break.

When $r_s = R$, the height h of the wave, is equal to $\frac{L}{\pi}$. This, therefore, is the theoretical relation between the height and wave length of the maximum possible wave. (See limiting value of $\frac{h}{L}$ deduced by Michell, Ch. IV.)

It is doubtful if this limit is reached except under very unusual circumstances. Out of many hundred observations which the writer has secured, the nearest approach to it was when the wave height was equal to $\frac{L}{5}$. This occurred in the case of steep superficial waves, formed by a wind which had sprung up suddenly and had blown steadily for about one hour with a velocity of about 25 miles per hour, the wave length being only about 15 feet and height about 3 feet.

It is not believed that storm waves of ordinary size are ever as steep as the one in the case last mentioned.

Lieutenant Paris, of the French navy, found the value of $\frac{L}{h}$ to be as follows:

In a light sea, $\frac{L}{h}=39$.

In a rough sea, $\frac{L}{h}=21$.

In a heavy sea, $\frac{L}{h}=19$.

Dr. Gerhard Schott, as a result of his observations, found that the proportion between height and length varied considerably according to the force of the wind, the values obtained being as follows:

Wind.		Value of $\frac{L}{h}$.		
Character.	Force (Beaufort scale).[a]	Maximum.	Minimum.	Mean.
Moderate	5	41	20	33
Strong	6 to 7	19	18	18
Storm	9 and more	21	13	17

[a] A scale of 12 terms, in which 1 represents light air and 12 a hurricane.

An examination of Table IX shows that out of 88 values of $\frac{L}{h}$ there given, 22 are included between the limits $\frac{L}{h}=15$ and $\frac{L}{h}=10$, the latter value indicating the steepest single wave recorded in the table.

The results given in this table agree fairly well with the values found by Lieutenant Paris and Doctor Schott, although, upon the whole, they indicate rather steeper waves than were encountered by these investigators.

The following table, taken from White's Manual of Naval Architecture, gives the approximate values for $\frac{L}{h}$, derived from the published French observations in all parts of the world. All observations for which the value $\frac{L}{h}$ was greater than 40 were omitted.

Length of waves.	Number of observations.	Length ÷ height.		
		Average.	Maximum.	Minimum.
100 feet and under	11	17	30	5
100 to 200 feet	55	20	40	9
200 to 300 feet	44	25	40	10
300 to 400 feet	36	27	40	17
400 to 500 feet	17	24	40	15
500 to 650 feet	16	23	40	17

Of the 650 observations upon waves in fresh and relatively shallow water, which are given in Table X, the value of $\frac{L}{h}$ in 235 cases varies between the limits 9.1 and 15.0, indicating, as was to be expected, steeper waves than are encountered on the ocean.

From what precedes, it seems certain that storm waves for which $\frac{L}{h}$ is less than 9 will rarely be encountered.

REDUCTION OF HEIGHT OF WAVES UNDER LEE OF A DETACHED BREAKWATER.

When waves are deflected in rear of a detached breakwater their height is reduced to an extent depending upon the distance traveled by the wave and the amount of deflection. Important as this subject is there appears to be no well-established formula, based upon extended observations, for determining accurately the decrease in height due to this deflection.

Mr. Thomas Stevenson found that the amount of reduction increased directly as the distance traversed, and as the square root of the number of degrees in the angle of deflection, but was careful to state that his conclusions are based upon a rather slender stock of facts, and expressed the hope that other observers would take up this important line of investigation.

REDUCTION IN HEIGHT OF WAVES ON PASSING INTO A CLOSE HARBOR.

Stevenson states that "when the piers are high enough to screen the inner area from the wind, where the depth is tolerably uniform, the width of entrance not very great in comparison with the section of the wave, and when the quay walls are vertical, or nearly so, and the distance not less than 50 feet from the mouth of the harbor to the place of observation, the following formula is applicable:"

$$X = h\frac{\sqrt{b}}{\sqrt{B}} - \frac{\left(h + h\frac{\sqrt{b}}{\sqrt{B}}\right)\sqrt[4]{D}}{50}; \qquad (19B),$$

in which

h=height of wave at entrance, in feet;

b=breadth of entrance, in feet;

B=breadth of harbor at place of observation, or, more accurately, length of arc, with radius D;

D=distance from mouth of harbor to place of observation, in feet;

X=height of reduced wave at place of observation, in feet.

He was of the opinion that the preceding formula was of general application in all close harbors where the entrance is of a direct and simple nature, and in which there is no recoil action, produced by walls or obstructions, to the shoreward motion of the waves.

The Duluth, Minn., ship canal and the adjoining harbor basin conform in a general way, though not strictly as regards all details, to the conditions described by Stevenson, and in 1902 the writer instituted observations at various points in the harbor, for the purpose of testing the applicability of the formula to the locality in question. The results of these observations were as follows:

REDUCTION OF WAVE HEIGHT IN DULUTH HARBOR, MINN.

Date.	Observation stations.	D.	B.	b.	h.	Height of wave.	
						Observed.	Computed.
1902.		*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*
April 21	3	1,200	3,351	300	9.9	1.17	1.45
21	4	1,300	2,813	300	9.9	2.25	1.66
21	5	2,600	3,857	300	9.9	2.50	.95
21	6	2,600	3,857	300	9.9	1.09	.95
21	7	2,930	4,142	300	9.9	2.25	.81
22	2	780	1,960	300	11.0	2.26	2.68
22	4	1,300	2,813	300	11.0	1.74	1.84
22	6	2,600	3,857	300	11.0	1.65	1.06
22	8	4,195	6,000	300	11.0	.45	.29
October 23	1	720	1,809	300	7.0	1.00	1.83
						16.27	13.52

OBSERVATIONS AT UPPER ENTRANCE, PORTAGE CANALS, LAKE SUPERIOR.

1902.							
September 11	A.	1,340	2,571	400	12.0	3.00	2.71

An inspection of the preceding table shows, as might be expected, that on April 21 the waves were practically of the same height at stations 4, 5, and 7, due to the fact that the harbor basin is quite unsymmetrical with respect to the axis of the canal (See Pl. III). Entering waves are prevented by bulkheaded wharves from expanding on the north side of the harbor basin, and consequently experience but little reduction while passing from station 4 to station 7. Annoyance from swells in the harbor during storms would have been almost entirely prevented if the city of Duluth had cut the canal farther to the south, thus permitting entering waves to expand freely before reaching the north side of the harbor.

TIDAL WAVES, WAVES PRODUCED BY EARTHQUAKES AND VOLCANIC ACTION, AND SEICHES.

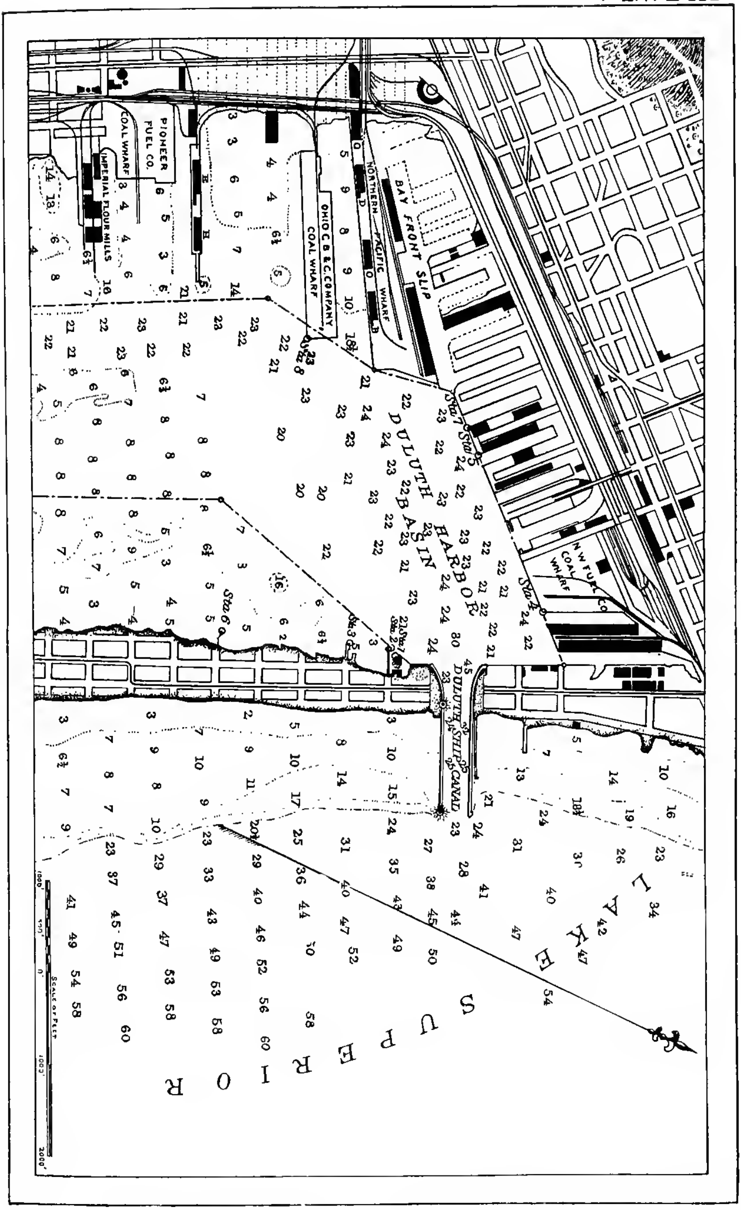
When a free tide wave passes into an arm of the sea, which decreases in width and in depth, the energy of the wave is gradually concentrated into a diminishing volume of water with the result that the height is increased, in some cases to a remarkable extent. A well-known instance, and one which is usually exaggerated, is the range of the tide at the head of the Bay of Fundy. The average range of spring tides, near the time of the moon's perigee, in Cumberland Basin, at the head of the Bay of Fundy, is 47.6 feet, while on the New England coast it is only about one-fourth as great.

The extreme difference in level between the lowest low water ever recorded at Cumberland Basin and the highest high water ever known is 53 feet, the latter record being obtained about twenty-four years before the former.

In Bristol Channel the rise of spring tides at the mouth is about 18 feet, at Swansea about 30 feet, and at Chepstow about 50 feet.

It is stated that at the time of the earthquake which destroyed the city of Lisbon in 1755 a wave 40 feet in height rolled in, followed by two others nearly as great in size, while at Cadiz the height of the wave was 60 feet. In the Antilles the sea rose 20 feet, the ordinary rise of tide rarely exceeding 2 feet.

Humboldt states that upon one occasion at Callao he noticed


LAKE SUPERIOR
DULUTH HARBOR BASIN
DULUTH SHIP CANAL
NORTHERN PACIFIC WHARF
BAY FRONT SLIP
OHIO C. B. & C. COMPANY COAL WHARF
PIONEER FUEL CO.
COAL WHARF
IMPERIAL FLOUR MILLS
N. W. FUEL CO. COAL WHARF
SCALE OF FEET

a number of waves 10 or 14 feet high in the midst of a dead calm. .

The noted eruption of the volcano Krakatoa, in August, 1883, destroyed all the towns and villages on the shores of Java and Sumatra nearest to the volcano. All boats and vessels on the same shores were destroyed, and 36,380 lives were lost. The tidal wave which followed was 50 feet in height when it struck the shores of Java and Sumatra. At Merak, on the Java coast, it was 135 feet in height. A man-of-war, lying off the Sumatra coast, was carried a mile and three-quarters inland, up a valley, and left in a forest 30 feet above sea level. The tidal wave extended to Colombo, 1,760 miles distant; to Bombay, 2,700 miles distant; to Cape Horn, 5,000 miles distant. This wave traveled at the rate of 350 miles per hour.

The fluctuations of water level known as *seiches*, which are so noticeable upon the Great Lakes, are usually formed as follows: When the wind blows with considerable velocity in a direction parallel to the longer axis of the lake, its effect is to lower the level of the water at one end of the lake and to raise it at the opposite end, thus disturbing the normal condition of hydrostatic equilibrium. When the velocity of the wind begins to decrease, the water tends to regain a condition of stable equilibrium by a series of oscillations about a line at right angles to the longer axis of the lake and about midway between the extreme ends, the oscillations sometimes continuing for three or four days.

Owing to its small depth and to its shape, the fluctuations in level are greater on Lake Erie than on any other of the Great Lakes.

During the severe storm of November 21, 1900, when the wind attained a velocity of 80 miles per hour at Buffalo, the lake level was raised 8.4 feet, falling again to its previous level, and completing the oscillation in about sixteen hours.

During this period the water level at Amherstberg, at the opposite end of the lake, fell 4.6 feet, the oscillation being completed in the same period, sixteen hours. The greatest simultaneous difference of level between the two ends of the lake during this period was 13.1 feet.

Seiches upon Lake Superior seldom have a greater range than 3 or 4 feet, and this difference in level has been known

to occur on some occasions in from ten to fifteen minutes. Many seiches on the latter lake can not be connected with known wind action, and their origin has been attributed to sudden local changes in atmospheric pressure. So far as the writer's own observations on Lake Superior extend it has not yet been clearly proven that they are due to this cause, investigation along this line being unsatisfactory on account of the few places on the lake where continuous barometer records are kept.

CHAPTER VI.

Velocity of Wave Propagation and of Orbital Motion of Particles.

Deep-water waves.—Theoretical velocity. Comparison of observed and computed wave velocities. Individual observations upon deep-water waves liable to some error.

Waves in shallow water.—Theoretical velocity. Comparison of observed and computed results at Peterhead, North Britain; North Beach, Fla., and on Lake Superior. Effect of diminishing depth upon wave length and velocity. Relation between wind and wave velocity. Velocity of positive and negative waves, tidal waves, and seiches. Effect of surface tension on the velocity of very small waves and ripples.

DEEP-WATER WAVES.

The theoretical velocity of propagation of a wave in deep water is equation (6).

$$v=\sqrt{\frac{gL}{2\pi}}=\sqrt{gR}=\frac{L}{t}=\frac{gt}{2\pi}=\sqrt{5.123L};$$

in which t = wave period, or interval of time, expressed in seconds, between the passage of consecutive wave crests, the other quantities being as previously explained.

The uniform velocity v' of a particle in its circular orbit of radius r is—

$$v'=\frac{gr}{v}=\frac{2\pi r}{t}=r\sqrt{\frac{2\pi g}{L}}=\frac{vr}{R}=r\sqrt{\frac{g}{R}}; \qquad (20).$$

The formulæ relating to wave motion are deduced under the assumption that this motion takes place in a perfect fluid, the pressure upon the surface of which is uniform. Water is not a perfect fluid, and its surface is not strictly a free surface, but rather a surface of separation between two fluids, one of which is liquid and incompressible and the other gaseous and compressible. Moreover, the latter is usually in motion with respect to the wave form, and as a consequence the assumed uniform pressure upon the surface does not

strictly exist, although the resulting error is probably very small.

For the reasons stated, it would scarcely be expected that observed velocities of deep-water waves, even if taken with the greatest accuracy, would be found to agree exactly with results computed by the formulæ, but, as will be shown hereafter, observations upon waves of this class, being taken from vessels, are more liable to unavoidable errors than are observations upon waves in shallow water, taken from piers, jetties, breakwaters, etc. as bases, where suitable arrangements can be made in advance for securing with greater accuracy the observations contemplated.

As the theoretical velocity of a deep-water wave depends only on the wave length, it is desirable to tabulate as many observations as practicable, in which both the wave length and velocity are given, for the purpose of comparing the observed velocity with that computed by equation (6).

The following table, in which the observations are arranged with reference to the observed wave lengths, has been prepared from Table IX, Chapter V:

TABLE XI.—*Comparison of observed and computed velocities of deep-water waves.*

Wave height. (1)	Wave length. (2)	Wave velocity (feet per second). Computed. (3)	Observed. (4)	Difference, columns (3) and (4).
24.6	1,121	75.8	77.3	− 1.5
46.0	765	62.5	46.4	+16.1
36.1	650	57.6	58.0	− 0.4
37.5	600	55.4	40.0	+15.4
13.1	571	54.1	57.1	− 3.0
43.0	559	53.5	47.7	+ 5.8
25.0	500	50.6	38.5	+12.1
4.0	500	50.6	50.0	+ 0.6
19.7	460	48.5	48.5	0.0
24.6	459	48.4	41.7	+ 6.7
14.4	459	48.4	45.9	+ 2.5
25.0	450	47.9	40.9	+ 7.0
26.2	428	46.8	48.2	− 1.4
11.5	426	46.6	42.6	+ 4.0
9.8	426	46.6	43.5	+ 3.1
24.0	400	45.2	33.3	+11.9
22.0	400	45.2	40.0	+ 5.2
21.0	400	45.2	44.4	+ 0.8
18.0	400	45.2	33.3	+11.9
8.0	400	45.2	40.0	+ 5.2

TABLE XI.—*Comparison of observed and computed velocities of deep-water waves*—Continued.

Wave height. (1)	Wave length. (2)	Wave velocity (feet per second). Computed. (3)	Observed. (4)	Difference, columns (3) and (4).
21.3	394	44.9	21.3	+23.6
14.8	394	44.9	49.2	− 4.3
32.0	380	44.1	43.6	+ 0.5
25.0	375	43.8	50.0	− 6.2
33.6	374	43.7	49.9	− 6.2
23.0	350	42.3	35.0	+ 7.3
16.0	350	42.3	38.9	+ 3.4
16.0	350	42.3	31.8	+10.5
21.0	328	40.9	41.0	− 0.1
14.8	328	40.9	41.0	− 0.1
6.6	328	41.0	44.6	− 3.6
18.0	318	40.3	42.4	− 2.1
5.0	314	40.0	38.1	+ 1.9
8.2	305	39.5	39.4	+ 0.1
18.0	300	39.1	37.5	+ 1.6
15.0	300	39.1	33.3	+ 5.8
12.0	276	37.5	55.2	−17.7
19.7	262	36.6	27.6	+ 9.0
8.2	262	36.6	43.7	− 7.1
5.0	261	36.5	33.9	+ 2.6
18.0	250	35.7	27.8	+ 7.9
15.0	250	35.7	27.8	+ 7.9
8.0	249	35.7	35.6	+ 0.1
10.5	209	32.7	31.2	+ 1.5
14.8	202	32.2	33.5	− 1.3
15.0	200	32.0	25.0	+ 7.0
13.1	193	31.5	28.9	+ 2.6
8.0	191	31.2	31.3	− 0.1
11.1	180	30.3	32.7	− 2.4
8.0	171	29.6	22.8	+ 6.8
11.0	167	29.2	24.6	+ 4.6
11.5	164	28.9	27.3	+ 1.6
2.6	162	28.8	31.2	− 2.4
14.0	160	28.6	26.7	+ 1.9
6.6	158	28.4	25.1	+ 3.3
15.0	150	27.7	21.4	+ 6.3
11.5	148	27.5	21.1	+ 6.4
9.8	148	27.5	26.4	+ 1.1
4.6	148	27.5	29.8	− 2.3
8.2	145	27.3	26.9	+ 0.4
3.9	134	26.2	26.9	− 0.7
8.2	131	25.9	20.2	+ 5.7
8.2	131	25.9	23.8	+ 2.1
7.5	131	25.9	21.7	+ 4.2
4.9	131	25.9	26.2	− 0.3
6.6	123	25.1	25.6	− 0.5
3.3	119	24.7	24.3	+ 0.4
8.9	115	24.2	19.2	+ 5.0

TABLE XI.—*Comparison of observed and computed velocities of deep-water waves*—Continued.

Wave height. (1)	Wave length. (2)	Wave velocity (feet per second). Computed. (3)	Observed. (4)	Difference, columns (3) and (4).
7.5	115	24.2	19.2	+ 5.0
4.6	115	24.2	24.0	+ 0.2
3.6	115	24.2	22.1	+ 2.1
2.6	108	23.5	23.6	− 0.1
8.0	100	22.6	14.3	+ 8.3
7.0	100	22.6	16.7	+ 5.9
6.0	100	22.6	16.7	+ 5.9
4.0	100	22.6	12.5	+10.1
6.6	98	22.4	16.3	+ 6.1
4.9	98	22.4	21.8	+ 0.6
4.3	82	20.5	20.5	0.0
6.0	80	20.2	16.0	+ 4.2
5.0	60	17.5	12.0	+ 5.5
4.0	50	16.0	10.0	+ 6.0
3.0	50	16.0	12.5	+ 3.5
2.0	40	14.3	10.0	+ 4.3
3.0	30	12.4	10.0	+ 2.4
85.0	....	2,995.6	2,737.9	385.3 {+321.5, − 63.8}

The observations in the preceding table were taken by 11 different sets of observers on 41 different dates, and in 62 out of a total of 85 observations the velocity computed by the formula, $v=\sqrt{5.123L}$, is greater than that observed, while in 23 instances it is less.

Upon inspection of the table it would seem that the individual discrepancies between observed and computed results are greater than they should be, but it should be remembered that observations upon deep-water waves are usually taken from vessels in motion, and that allowance must be made for the speed of the vessel and its direction with respect to that of wave travel.

The question of proper allowance for the speed of the vessel is complicated by the fact that when the water is rough the speed is seldom uniform, being greatest when the vessel is on the anterior slope of a wave traveling in the same direction as the vessel itself, and least when on the anterior slope of one traveling in the opposite direction.

It is therefore to be expected that individual observations

taken under such circumstances should show considerable discrepancies when compared with computed results.

WAVES IN SHALLOW WATER.

The theoretical velocity of propagation of a wave in shallow water is, equation (16)—

$$v=\sqrt{\frac{b_s}{a_s}.\frac{gL}{2\pi}}=\sqrt{\frac{b_s}{a_s}\cdot gR}=\sqrt{\frac{b_s}{a_s}5.123L}.$$

The ratio $\frac{b_s}{a_s}$ is given in Table III, and is shown graphically in Plate I.

The angular velocity of a particle m in its elliptical orbit is not constant, as in the case of the circular orbits of the deep-water wave, but is such that it will always be found in the vertical line Bm' passing through a supposed particle m', moving with constant angular velocity on the circumference of the circle whose center is c and whose radius is the semi-major axis of the ellipse.

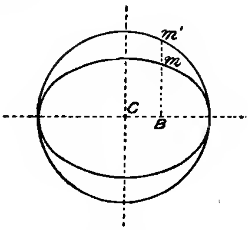


Fig. 10.

It therefore follows that the orbital velocity v'' of a particle m is least at and near mid height and greatest at and near the highest and lowest points of the orbit, and that its maximum velocity is given by the equation.

$$v''=\frac{2\pi a'}{t}=\frac{v a'}{R}=a'\sqrt{\frac{b_s}{a_s}.\frac{g}{R}}=a'\sqrt{\frac{b_s}{a_s}.\frac{2\pi g}{L}}; \qquad (21).$$

It will be seen from equation (16) that the velocity of wave propagation depends upon the wave length and upon the ratio $\frac{b_s}{a_s}$ of the semi axes of the surface orbit.

This ratio is itself dependent upon the wave height, $h=2b_s$, the depth d_0, measured from the mid height of the wave, and the wave length L.

In order, therefore, to compare observed velocities with those computed by equation (16) these three quantities and the corresponding velocity must have been noted.

This appears seldom to have been done, and consequently it has not been practicable to form a table of observations taken by different observers at various localities for the purpose of comparing observed and computed velocities, as was done in the case of the deep-water wave.

Observations at Peterhead, North Britain, 1900.—Observations upon waves at Peterhead, North Britain, during the severe storm of February 15–16, 1900, were discussed by Mr. William Shield in a paper read before the Institution of Civil Engineers of Great Britain, in which it was stated that during the storm the wind fluctuated between 50 and 89 miles per hour, and that on the afternoon of February 16, when the wind had somewhat abated, observations were taken as the waves ran into the bay between the end of the breakwaters and the opposite shore, the depth at the point of observation being from 60 to 63 feet.

The waves were irregular in respect to height, length, and period, but wave after wave passed with unbroken crest fully 22 feet 6 inches above still-water level. The periods of the waves varied between 13 and 17 seconds, and the lengths were estimated to be between 500 and 700 feet, but the author had no means of determining this accurately.

Comparing the data given with similar data secured from observations taken on Lake Superior, in which the portion of the wave above still-water level, the wave length, and the depth bore the same relations as in the case of the observations at Peterhead, it was found that about 64 per cent of the wave height was probably above still-water level and about 36 per cent below, or the observed waves were probably about 35 feet in height.

If we take the mean estimated wave length, 600 feet, and the mean wave period, 15 seconds, and assume that 64 per cent of the wave height was above the level of still water, we have from equation (16), $v=\sqrt{5.123\times0.6\times600}=42.9$ feet per second.

The corresponding observed velocity was 40 feet per second, a satisfactory degree of accordance when it is remembered that the wave length was estimated and not measured.

At Peterhead, in November, 1888, Mr. Shield measured waves 26 feet in height and 500 feet in length, which traveled with a velocity of 41 feet per second in water 7 to 8 fathoms in depth.

Assuming in this and in the following case that two-thirds of the wave height was above the still-water level, and computing the theoretical velocity by equation (16), we have v=37.6 feet per second.

Mr. Shield, from a staging in Algoa Bay, in water 24 feet in depth, measured waves 10 feet in height and 200 feet in length, which traveled with a velocity of 20 feet per second. The corresponding wind velocity was 58 miles per hour.

The value of v computed from equation (16) is in this case 26.1 feet per second, a result which differs considerably from the velocity actually observed.

Observations at North Beach, Florida, 1890–91.—Observations upon the velocity of ocean waves immediately before breaking were made by the writer at North Beach, St. Augustine, Fla., between January, 1890, and May, 1891. In all, nearly 100 observations were made upon waves varying in height from 2 inches to 7 feet.

Eighty-four of the observations were made during calm weather, or when only moderate winds prevailed, and with the idea of eliminating any possible effect of wind, these observations only have been selected and are given in the following table:

Table showing velocity of waves in shallow water immediately before breaking, North Beach, St. Augustine, Fla., 1890–91.

Wave height. (1)	Wave length. (2)	$\frac{d_0}{L}$ (3)	$\frac{b_s}{a_s}$ (4)	Computed velocity. $v=\sqrt{\frac{b_s}{a_s}5.123L}$ (5)	Observed velocity. (6)	Difference, columns (5) and (6)	Number of observations.
Inches.	*Feet.*						
2	2.0	0.070	0.425	2.1	1.7	+ 0.4	2
3	4.0	.078	.466	3.1	2.3	+ 0.8	2
4	6.0	.073	.440	3.7	2.5	+ 1.2	2
6	10.0	.063	.388	4.5	2.8	+ 1.7	4
8	14.0	.060	.372	5.2	4.0	+ 1.2	2
10	20.0	.053	.325	5.8	5.0	+ 0.8	2

Table showing velocity of waves in shallow water immediately before breaking; North Beach, St. Augustine, Fla., 1890–91—Continued.

Wave height. (1)	Wave length. (2)	$\frac{d_o}{L}$ (3)	$\frac{b_s}{a_s}$ (4)	Computed velocity. $v=\sqrt{\frac{b_s}{a_s}5.123L}$ (5)	Observed velocity. (6)	Difference, columns (5) and (6)	Number of observation.
Inches.	*Feet.*						
1.0	23.0	.060	.372	6.6	5.4	+ 1.2	5
1.25	27.0	.058	.360	7.1	6.8	+ 0.3	5
1.5	30.0	.067	.412	8.0	7.0	+ 1.0	3
1.7	35.0	.069	.420	8.7	8.0	+ 0.7	1
2.0	46.0	.065	.400	9.7	8.4	+ 1.3	4
2.25	50.0	.066	.406	10.2	9.4	+ 0.8	4
2.5	60.0	.062	.382	10.8	9.4	+ 1.4	10
2.75	70.0	.059	.365	11.4	10.3	+ 1.1	5
3.00	75.0	.063	.390	12.2	11.7	+ 0.5	6
4.0	82.0	.077	.460	13.9	12.2	+ 1.7	10
4.25	85.0	.074	.445	13.9	13.0	+ 0.9	2
4.5	90.0	.074	.445	14.3	14.0	+ 0.3	4
5.0	120.0	.060	.372	15.1	15.2	− 0.1	8
6.0	150.0	.060	.372	16.9	18.2	− 1.3	2
7.0	160.0	.067	.412	18.4	21.5	− 3.1	1
Total.	....	....	....	201.6	188.8	21.8 +17.3 − 4.5	84

An inspection of this table shows that the velocity computed by equation (16) is greater than the observed in 18 out of 21 instances.

The facilities for securing accurate observations at North Beach were rather crude, and consequently the accordance between observed and computed results is probably as close as was to be expected under the circumstances.

Observations on Lake Superior, 1901 and 1902.—During the seasons of 1901 and 1902, whenever opportunity occurred, observations were made under the direction of the writer near the outer end of the Duluth, Minn., Ship Canal, and in Lake Superior near the canal. The average mean depth at the point where the observations in the canal were taken was about 26 feet, and the clear width between the straight and parallel canal piers about 300 feet. The piers project about 1,000 feet out into Lake Superior, and their direction is such that during northeast storms the waves run squarely into the mouth of the canal and are readily observed from either pier. To the north and south of the canal the lake bottom slopes

to the shore from a depth of about 25 feet to 0 feet in a distance of about 1,100 feet. As the direction of wave travel is parallel to the piers, simultaneous observations could be taken outside of the piers and in the canal itself.

Velocity observations in the canal were usually taken between stations 300 feet apart, the outer station being 150 feet from the outer end of the piers.

In all, 631 observations were taken upon waves of heights varying from 2 to 23 feet, of wave lengths varying from 45 to 275 feet, and of velocity varying from 9.1 to 33.3 feet per second. The mean depth varied from 3.3 to 27 feet.

The results of these observations are given in the following table:

TABLE XII.—*Comparison of observed and computed wave velocities, Lake Superior, 1901–2.*

SEC. I. OBSERVATIONS IN DULUTH SHIP CANAL, MEAN DEPTH 24.7 TO 27 FEET.

Date.	Wave velocities (feet per second).						Sum of individual velocities.		Number of observations.
	Observed.			Computed.					
	Max.	Min.	Mean.	Max.	Min.	Mean.	Observed.	Computed.	
1901.									
May 23	32.0	24.7	27.1	27.9	25.5	26.7	1,058.4	1,039.7	39
24	25.0	20.0	22.3	24.9	24.0	24.6	66.8	73.7	3
June 14	25.5	22.5	25.3	24.3	23.4	24.1	506.8	482.4	20
15	21.8	20.4	21.4	24.5	23.2	23.8	450.2	500.8	21
July 24	24.2	19.8	21.1	25.0	22.5	23.8	844.0	952.0	40
Aug. 9	24.2	16.7	21.6	24.3	21.8	22.9	1,060.2	1,122.6	49
Sept. 24	33.2	25.0	27.7	28.3	26.4	27.2	2,213.0	2,174.5	80
Oct. 9	23.7	21.5	22.9	24.9	24.3	24.6	687.6	739.2	30
10	23.2	19.1	22.7	25.7	24.3	24.9	453.6	498.8	20
Nov. 6	30.8	26.7	28.7	25.2	24.0	24.7	574.2	493.4	20
22	27.2	23.6	25.3	26.0	24.5	25.6	783.7	538.2	21
	......	......	......	......	......	......	8,698.5	8,615.3	343
1902.									
Apr. 21	30.0	21.1	23.7	27.0	22.0	24.7	1,467.0	1,532.3	62
22	33.3	23.1	28.5	28.4	21.9	26.8	626.4	590.0	22
23	20.0	20.0	20.0	24.9	24.9	24.9	80.0	99.6	4
May 1	25.0	21.0	23.4	25.3	21.9	23.3	234.0	232.7	10
18	18.2	18.2	18.2	18.8	18.8	18.8	36.4	37.6	2
20	25.0	20.0	22.9	27.0	22.8	25.4	366.3	406.3	16
31	21.0	21.0	21.0	21.8	21.8	21.8	84.0	87.2	4
June 3	22.2	22.2	22.2	23.7	23.7	23.7	88.8	94.8	4
4	18.2	18.2	18.2	18.7	18.7	18.7	54.6	56.1	3
5	21.0	21.0	21.0	21.9	21.9	21.9	63.0	65.7	3
Aug. 15	18.2	18.2	18.2	18.8	18.8	18.8	54.6	56.4	3
18	22.6	22.6	22.6	24.1	24.1	24.1	90.4	96.4	4

TABLE XII.—*Comparison of observed and computed wave velocities, Lake Superior, 1901–2*—Continued.

SEC. I. OBSERVATION IN DULUTH SHIP CANAL, ETC.—Continued.

Date.	Wave velocities (feet per second).						Sum of individual velocities.		Number of observations.
	Observed.			Computed.					
	Max.	Min.	Mean.	Max.	Min.	Mean.	Observed.	Computed.	
1902.									
Oct. 13	26.0	26.0	26.0	24.7	24.7	24.7	104.0	98.8	4
23	30.0	21.4	25.0	27.1	24.0	25.3	225.4	228.1	9
25	30.0	21.4	25.7	27.2	22.5	25.7	282.6	282.6	11
26	27.3	27.3	27.3	25.7	25.7	25.7	27.3	25.7	1
Nov. 12	30.0	21.4	26.0	28.2	25.7	27.1	598.4	624.3	23
13	25.0	25.0	25.0	27.1	26.5	27.0	125.0	134.9	5
Total Sec. I							13,306.7	13,364.8	533

SEC. II. OBSERVATIONS IN LAKE SUPERIOR, MEAN DEPTHS 3.3 TO 18.7 FEET.

Date.	Observed Max.	Observed Min.	Observed Mean.	Computed Max.	Computed Min.	Computed Mean.	Sum Observed.	Sum Computed.	Number of observations.
1902.									
Apr. 22	25.0	19.0	21.9	24.3	20.3	22.2	459.0	465.3	21
23	12.9	11.2	11.9	13.7	11.9	12.7	83.5	88.7	7
26	16.0	15.0	15.6	15.5	14.6	15.1	78.0	75.4	5
May 20	24.0	15.0	19.0	23.4	21.1	19.0	247.5	285.1	13
31	12.5	9.1	10.2	11.4	10.1	10.5	61.4	63.2	6
June 3	10.5	10.5	10.5	10.9	10.9	10.9	31.5	32.7	3
4	12.5	12.5	12.5	13.1	13.1	13.1	25.0	26.2	2
5	18.2	10.4	13.5	15.0	12.0	13.6	229.6	230.7	17
Oct. 23	24.1	15.4	18.3	21.4	19.0	20.4	274.8	306.4	15
25	24.1	15.4	20.1	20.8	19.4	20.1	180.7	180.8	9
Total Sec. II							1,671.0	1,754.5	98
Totals, Secs. I and II							14,977.7	15,119.3	631

The individual velocities, which have been aggregated in the ninth column, were all computed by equation (16).

The observations in Section I of the preceding table, 533 in number, were taken in the outer portion of the Duluth ship canal, where there are excellent facilities for securing accurate observations. Although the corresponding observed and computed velocities show both plus and minus discrepancies in individual instances, yet it will be noted from the table that the sum of the 533 observed velocities in Section I differs from that of the corresponding computed velocities by less than one-half of 1 per cent.

The observations in Section II of the table could not be taken with the same degree of accuracy as those in Section I, owing to inferior facilities for observing, yet the sum of the

98 observed velocities in Section II differs from that of the corresponding computed velocities by only about 5 per cent.

Considering both sections of the table, embracing 631 observations in all, the sum of the individual observed velocities differs from that of the computed velocities by less than one per cent, a result which must be regarded as satisfactory and as confirming the reliability of equation (16), especially when the wide range in wave heights, lengths, and velocities is taken into consideration.

Effect of decreasing depth upon wave length and velocity.—During the progress of wave observations in the vicinity of the Duluth canal, in 1902, it was considered desirable to note the effect produced upon wave length and velocity by a gradual decrease in depth.

Observations for length and velocity were taken just as the waves entered the canal and compared with similar observations upon corresponding waves in shoaler water in the lake.

The results of these observations are given in the following table.

The velocities in the tenth column were computed by the following empirical formula deduced by the writer:

$$v_1 = 0.9v\sqrt[4]{\frac{d_1}{d}}; \qquad (16A).$$

in which v and v_1 are the velocities of the same wave for the depth d and d_1, the depth d being greater and the water shoaling from d to d_1.

It is not probable that the preceding formula would apply where the depth d is much greater than $\frac{L}{3}$, nor where the slope of the bottom is much steeper than in the cases embraced in the table.

The "mean depth of water" given in columns 4 and 8 refers to the mean depth over which the observations for wave length and velocity extended, generally embracing a distance of about 200 feet, the depth increasing lakeward.

When a wave reached a depth in the lake at any point "*b*," near the mouth of the canal, equal to the mean depth in the canal, the portion of the wave entering the canal traveled on with practically undiminished velocity, while the portions of the same wave in the lake on each side of the canal gradually

decreased in wave length and velocity, owing to the effects of diminishing depth. The "distance traveled by wave," given in column 9, is the distance from the point "*b*" to the mid-point of observation in shallower water nearer shore.

TABLE XIII.—*Comparison of lengths and velocities of waves in the Duluth Ship Canal with those of corresponding waves in shallower water in the lake adjacent to the canal, arranged according to depths in the lake.*

Duluth Ship Canal.				Lake Superior, near canal.					
Wave.			Mean depth of water. d	Wave.			Mean depth of water. d_1	Distance traveled by wave.	Computed velocity. V_1
Height. h	Length. L	Velocity. v		Height. h_1	Length. L_1	Velocity. v_1			
....	209	28.5	25.8	8.6	174	22.4	18.7	316	23.6
....	168	27.1	25.9	....	165	21.3	18.4	316	22.4
....	159	22.9	26.5	7.5	159	19.0	15.7	408	18.1
....	124	23.4	26.0	6.6	....	18.7	15.3	408	18.5
....	164	25.7	26.7	7.6	110	20.1	14.0	493	19.9
....	162	25.0	26.5	6.9	140	18.3	13.7	493	19.1
....	209	28.5	25.8	7.4	162	21.6	13.5	408	21.8
....	160	21.1	26.0	6.2	77	....	12.0	800	....
....	100	21.0	26.6	3.3	69	14.6	6.9	1,200	13.4
....	70	18.2	25.7	3.0	50	12.5	5.7	1,175	11.3
....	165	20.0	24.7	3.0	75	12.9	5.6	1,120	12.4
....	100	21.0	26.6	3.0	50	13.3	5.6	1,220	12.9
....	100	21.0	26.6	3.0	60	12.8	5.1	1,136	12.5
....	100	21.0	26.0	2.5	60	12.5	4.2	1,248	11.9
....	165	20.0	24.7	2.6	65	11.2	4.0	1,208	11.3
....	130	22.2	25.8	2.5	50	10.5	3.3	1,270	12.0
....	100	21.0	26.0	2.0	45	9.1	3.3	1,276	11.3
Total.	....	....	....	....	....	250.8	....	....	252.4

Relation between wind velocity and that of wave propagation.—It has been noticed that storms at sea sometimes signal their approach by a rising swell entirely out of proportion to the wind direction and velocity at the place of observation. This fact has been taken to indicate that the velocity of a wave may exceed that of the wind which generates it. The writer's observations do not confirm this conclusion, and he is of the opinion that the undisputed fact that waves at times precede the wind may be explained satisfactorily by the known laws governing the movement of storms.

In severe storms, while the storm center itself may move approximately in a straight line, the actual wind direction is

along a line curving around this center. The waves generated by the wind at any point travel on a tangent to the curve of wind direction, while the wind continues along the curved path around the storm center, the latter moving relatively slowly in comparison with the velocity of the wind around it.

It will thus be seen that these tangential waves may appear in advance of a storm, and yet travel with less velocity than that of the wind which generates them.

Dr. Gerhard Schott, as a result of his observations upon ten occasions in 1891–92, found that the wind velocity always exceeded that of the waves. Denoting the velocity of the wind by v_w, and that of the wave by v, Doctor Schott found that the ratio $\frac{v_w}{v}$ varied between the limits 1.17 and 1.51, the mean value being 1.32.

The wind velocity used in obtaining this ratio was determined from the wind force noted every two hours by Beaufort scale. Although the wind velocities were not measured, Doctor Schott is of the opinion that they may be fully relied upon, as in sailing vessels like those in which he traveled, the force of the wind is carefully estimated, since it governs the amount of sail to be carried.

Lieutenant Paris, of the French navy, found that in a very heavy sea $\frac{v_w}{v}=1.66$; in a heavy sea, 1.43, and in a rough sea, 1.07.

The large velocity which Lieutenant Paris gives for waves in moderate winds causes the ratio $\frac{v_w}{v}$ to become less than unity in these cases.

In 1901–2 the writer paid particular attention to a comparison of wind and wave velocities at Duluth, Minn., the results of which are given in the following table. The wave velocity was observed in the outer end of the Duluth Ship Canal in water of a mean depth of about 26 feet, a depth which undoubtedly caused a decrease in the velocity with which the corresponding wave had traveled in deep water. Due to this, the ratio $\frac{v_w}{v}$ is larger than it would be under the same conditions out on the lake.

TABLE XIV.—*Comparison of velocity of waves in the Duluth Canal with corresponding wind velocity at station 1.25 miles distant, wind blowing in the direction of wave travel.*

Date.	Mean wave velocity (feet per second).		Hourly wind velocity (feet per second).			Ratio of wind velocity during observations to that of waves. $\frac{V_w}{V}$
	Observed.	Computed.	During wave observations.	Maximum during 12 hours preceding.	Mean for 12 hours preceding.	
1901.						
May 23	27.1	26.7	48.2	80.7	57.5	1.78
June 14	25.3	24.1	49.1	55.9	29.6	1.94
Sept. 24	27.7	27.2	67.6	70.9	60.6	2.44
Oct. 9	22.9	24.6	42.2	49.1	28.6	1.84
Nov. 6	28.7	24.7	62.6	72.5	32.0	2.18
22	25.3	25.6	35.3	38.7	34.4	1.40
1902.						
Apr. 21	23.7	24.7	50.7	57.6	50.1	2.14
22	28.5	26.8	64.4	64.4	53.7	2.26
May 1	23.4	23.3	55.0	60.9	49.7	2.35
20	22.9	25.4	49.1	55.9	49.1	2.14
Aug. 15	18.2	18.8	29.0	31.8	18.8	1.59
Oct. 23	25.0	25.3	38.7	47.5	42.2	1.55
25	25.7	25.7	59.4	62.6	52.6	2.31
Nov. 12	26.0	27.1	56.7	57.6	38.8	2.18
Total	350.4	350.0	708.0	806.1	597.7	28.10
Mean	25.0	25.0	50.6	57.6	42.7	2.01

NOTE.—The wind velocities employed are the corrected velocities, as determined by the United States Weather Bureau, by means of a Robinson's anemometer located on the edge of the high bluff in rear of the city of Duluth; the wind velocity at this point being believed to approximate most closely to that out on the open lake.

An inspection of the table shows that in no case during a storm did the velocity of the waves equal that of the wind at the time of observation.

On several occasions when the storm had been markedly subsiding wave velocities in excess of that of the wind were observed, but these were so clearly the result of previous higher wind velocities that they form no exception to the fact previously stated, as it is well known that swells sometimes run in for many hours after the wind causing them has entirely subsided. For example, at 11 a. m., on November 13, 1902, "dead swells," 10 to 14 feet in height and 180 to 200 feet in wave length, were running in the mouth of the Duluth Ship Canal, although the northeast wind which caused them had entirely subsided for about eight hours previously.

Owing to the relatively small size of Lake Superior as compared with the ocean, the approach of a storm is rarely signaled by a preceding swell.

On one occasion, in December, 1902, a very heavy swell was observed at the Duluth Canal, no storm wind preceding, accompanying, or following it. This swell was evidently produced by a distant storm which failed to reach Duluth.

Rankine states that long waves, or waves in water that is very shallow compared with the length of the wave, have a velocity nearly independent of the wave length, and nearly equal to that acquired by a heavy body falling freely through a height equal to half the still-water depth, plus three-fourths of the wave height, or

$$v = \sqrt{g(d + \tfrac{3}{4} h)}. \qquad (22)$$

Since in shallow water about three-fourths of the wave height is usually above still-water level, the preceding value of v is that acquired by a heavy body falling freely through a height equal to half the depth, measured from the crest of the wave.

The writer has applied this formula to several hundred observations upon shallow-water waves, taken at North Beach, Fla., and on Lake Superior, and has found that it almost invariably gives results considerably in excess of the observed velocities.

Positive and negative waves in water of finite depth.—Mr. Scott Russell determined experimentally that the velocity of "the great primary wave of translation," or positive wave, in channels of uniform depth, was the same as that due to the action of gravity on a heavy body falling freely through a vertical distance equal to half the depth, measured from the top of the wave, but that the velocity of the negative wave, in considerable depths, was sensibly less than that due to gravity through half the depth, measured from the lowest point of the wave.

Bazin's experiments in 1859 on positive and negative waves in shallow water confirmed Scott Russell's observations with respect to the velocity of the positive wave. In the case of the negative wave, Bazin found that the velocity was nearly the same as that acquired by a heavy body falling freely

through a height equal to half the depth of water measured from the lowest point of the wave.

Tidal waves and seiches.—Tidal waves, waves due to earthquakes or volcanic eruptions, and seiches, are waves of great length, which generally make their effect felt in water of small depth compared with the length of the wave.

Prof. G. B. Airy has shown that when the length of a wave is not less than a thousand times the depth of the water, the velocity of the wave depends only on the depth, and is the same as that which a free body would acquire by falling from rest, under the action of gravity, through a height equal to half the depth of the water.

In such waves the orbits of the particles are very long and flat ellipses, the major axes of which are nearly equal at all depths, and the minor axes nearly proportional to the mean heights of the particles above the bottom. A particle on the bottom moves backward and forward along a horizontal line.

The tide wave of the Atlantic travels over 90° of latitude in twelve hours, or at the rate of about 520 miles per hour, corresponding to a depth of about 18,000 feet.

In the North Sea this wave covers less than 6° of latitude in nine hours, or at the rate of about 45 miles per hour, a velocity indicating a mean depth of about 140 feet.

The crest of the free-tide wave travels up the broad, lake-like expanse of the St. Johns River, Florida, between Jacksonville and Palatka, a distance of 64 miles, in about five hours and forty minutes, or at the rate of 11.3 miles per hour, a velocity corresponding to a depth of about 9 feet.

The wave in the Pacific Ocean caused by the great earthquake of 1868 was estimated to have traveled at the rate of 300 to 400 miles per hour.

That formed in the Straits of Sunda in 1883 by the eruption of the volcano Krakatoa reached the coast of France with a velocity estimated at 300 miles per hour.

The seiches in Lake Erie ordinarily travel with a velocity of about 34 miles per hour, corresponding to a depth of about 80 feet, which is believed to be somewhat greater than the mean depth of the lake along the direction of oscillation.

Effect of surface tension upon the velocity of very short waves and ripples.—In 1848 Stokes pointed out that surface tension in liquids should be considered in finding the pressure of the

free surface, but this appears not to have been done until about 1871.

Prof. P. G. Tait shows that in the case of oscillatory waves of short length, if the depth is infinitely great, as compared with the wave lengths, the velocity is—

$$v^2=\frac{gL}{2\pi}+\frac{2\pi T}{L\rho}; \tag{23}$$

in which T denotes the surface tension and ρ the density of the liquid. From this equation it is seen that in all cases the velocity is increased by surface tension, and more proportionately as the wave length is shorter.

As the wave velocity would increase with increase of wave length if gravity alone acted, and with decrease of wave length if surface tension alone acted, it is evident that there must be a minimum velocity corresponding to some definite wave length when, as is always the case, both act.

When v^2 is a minimum, the corresponding value of $L=2\pi\sqrt{\frac{T}{g\rho}}$; v^2 being $=2\sqrt{\frac{gT}{\rho}}$. With water as the fluid, L is then equal to 0.68 inches and $v=0.76$ foot seconds. This represents the slowest moving oscillatory wave, and one which is sometimes considered as marking the theoretical limit between waves proper and ripples.

The preceding deduction, while interesting from a theoretical point of view, is of little practical value as regards marine constructions on exposed sites, for the waves there encountered are of such dimensions that the effects of surface tension are inappreciable.

CHAPTER VII.

Per cent of wave height above water level.—Why a knowledge of this relation is important to engineers. Observations by Thomas Stevenson. Observations at North Beach, Florida, and on Lake Superior.

Depth in which waves break.—Importance of subject. Maximum depth in which ocean waves break. Observations by J. Scott Russell, Thomas Stevenson, and Henri Bazin. Observations at North Beach, Florida, and on Lake Superior.

PER CENT OF WAVE HEIGHT ABOVE WATER LEVEL.

For deep-water waves the theoretical relation of the crest of the wave to the undisturbed water level is given by equation (5), Chapter III.

For ordinary waves in shallow water this equation gives results too small.

The question is of importance to the engineer, as indicating the height to which a structure may be subjected to direct wave attack, or to the attack of heavy bodies floating on the water. A knowledge of this relation would be necessary, for example, in fixing the minimum allowable height for the deck of a wharf or pier supported upon open pile work, or in determining the height of a parapet wall intended to prevent waves running parallel with a pier from washing over it. In fact, few marine works at exposed sites are constructed and maintained without an appreciation of the value of a knowledge of the elevation of the wave crest with respect to the undisturbed water level. Especially is this the case in harbors like those on Lake Superior, where numbers of floating logs and blocks of ice 2 or 3 feet in thickness may act as battering rams at any point on a work from the vicinity of the wave hollow to its crest.

Few observations appear to have been made for the purpose of determining the relation under discussion, and, in consequence, it is often erroneously assumed that the true mean level, or undisturbed water level, is at the mid height of the wave, an assumption which, especially in case of waves in shallow water, is far from correct.

Mr. Thomas Stevenson states that at Wick Bay observa-

tions upon some of the large waves during storms showed that about two-thirds of the height of the wave was above the undisturbed water level and about one-third below.

At the mouth of the Columbia River, in water about 26 feet in depth, waves have been observed, the crests of which were about 15 feet above the undisturbed water level. The height of these waves was not noted, but it is fair to assume, as will subsequently be shown, that they probably did not differ greatly from 22 or 23 feet.

Observations at North Beach, St. Augustine., Fla., 1890–91.—At North Beach, St. Augustine, Fla., in 1890–91, the writer made careful observations upon 45 waves, varying in height from 2.5 to 6 feet, for the purpose of determining the elevation of the crest of the wave, immediately before breaking, with respect to the true mean level of the water at the time.

The greater number of the observations were made in calm weather upon waves due to very regular swells resulting from distant disturbances.

The bottom at the locality where the observations were taken was of fine sand and very smooth, hard, and regular, with a uniform slope of about $\frac{1}{100}$.

Denoting the wave height by h and the reference of the crest above the undisturbed water level by a, it was found that a varied between the limits $a=0.67h$ and $a=0.89h$, the mean value being $a=0.76h$.

With a gentle slope of the bottom, or an opposed wind, a was increased; while with a steeper slope, or with a wind in the direction of wave travel, a was decreased.

Observations on Lake Superior, 1901–2.—During the season of 1901–2, a large number of observations were made under the supervision of the writer upon waves in the outer end of the Duluth Ship Canal and in Lake Superior near the canal, with the view of determining the relation, if any, which existed between the wave height and the elevation of the wave crest above still-water level.

In all, 789 observations were secured, of which 616 were taken in the Duluth Ship Canal, in water of a mean depth of 26 feet, upon waves which passed on without breaking, and 173 were taken in Lake Superior upon waves traveling in shoaling water and breaking after passing from 50 to 100 feet beyond the points at which observations were taken.

The results of these observations are given in the following table:

TABLE XV.—*Per cent of wave height above still-water level, arranged according to wave heights.*

SEC. I. DULUTH SHIP CANAL, 1901-2.

Mean wave height.	Mean wave length.	Elevation of wave crest above still-water level. Observed.	Computed.	Expressed in per cent of wave height. Observed.	Computed.	Number of observations.	Mean depth of water.
Feet.	*Feet.*	*Feet.*	*Feet.*				*Feet.*
1.0	115	0.5	0.51	0.50	0.51	8	26.0
2.0	118	1.1	1.07	.54	.53	11	26.0
2.5	70	1.3	1.43	.52	.57	2	26.0
3.0	130	1.7	1.64	.58	.55	8	26.0
4.0	125	2.2	2.26	.55	.56	13	26.0
4.5	128	2.8	2.57	.62	.57	2	26.0
5.0	132	2.7	2.88	.55	.58	30	26.0
6.0	128	3.5	3.56	.58	.59	49	26.0
6.5	120	4.2	3.95	.65	.61	2	26.0
7.0	140	4.5	4.20	.64	.60	52	26.0
7.5	158	4.9	4.46	.66	.60	6	26.0
8.0	146	5.1	4.88	.64	.61	55	26.0
8.5	152	5.4	5.20	.64	.61	8	26.0
9.0	153	5.9	5.56	.66	.62	60	26.0
9.5	117	6.0	6.29	.63	.66	3	26.0
10.0	147	6.4	6.36	.64	.64	56	26.0
10.5	158	7.2	6.65	.68	.63	30	26.0
11.0	154	7.4	7.07	.67	.64	45	26.0
11.5	164	8.1	7.36	.70	.64	7	26.0
12.0	173	8.2	7.66	.68	.64	43	26.0
12.5	165	9.0	8.15	.72	.65	4	26.0
13.0	183	8.6	8.35	.66	.64	47	26.0
13.5	204	9.0	8.54	.67	.63	5	26.0
14.0	175	9.1	9.24	.65	.66	13	26.0
14.5	160	10.5	9.88	.72	.68	1	26.0
15.0	205	9.7	9.70	.65	.65	9	26.0
16.0	231	10.0	10.22	.63	.64	10	26.0
17.0	202	11.3	11.36	.66	.67	8	26.0
18.0	210	11.2	12.90	.62	.67	5	26.0
19.0	210	12.2	12.94	.64	.68	5	26.0
20.0	210	13.0	13.81	.65	.69	10	26.0
21.0	210	14.0	14.70	.67	.70	3	26.0
22.0	210	14.0	15.61	.64	.71	5	26.0
23.0	210	15.0	16.54	.65	.72	1	26.0
Total ..	....	....	....	....	....	616	....
Mean ..	....	....	....	.637	.620	....	....

TABLE XV.—*Per cent of wave height above still-water level, arranged according to wave heights*—Continued.

SEC. II. LAKE SUPERIOR NEAR DULUTH SHIP CANAL, 1902; OBSERVATIONS UPON WAVES JUST BEFORE BREAKING.

Mean wave height.	Mean wave length.	Elevation of wave crest above still-water level.		Ratio of wave length to height. $\frac{L}{h}$	Number of observations.	Mean depth of water.
		Feet.	Per cent of wave height.			
Feet.	*Feet.*					*Feet.*
2.0	....	1.9	0.95	....	3	3.3-18.7
2.5	59.0	1.8	.74	23.0	5	3.3-18.7
3.0	65.0	2.3	.77	22.0	10	3.3-18.7
3.5	65.0	2.6	.74	16.0	8	3.3-18.7
4.0	120.0	2.9	.73	30.0	9	3.3-18.7
4.5	90.0	3.5	.78	20.0	2	3.3-18.7
5.0	156.0	3.3	.67	31.0	13	3.3-18.7
5.5	144.0	3.8	.69	26.0	4	3.3-18.7
6.0	145.0	4.3	.71	24.0	18	3.3-18.7
6.5	124.0	4.6	.70	19.0	7	3.3-18.7
7.0	147.0	5.2	.74	21.0	27	3.3-18.7
7.5	150.0	5.1	.68	20.0	5	3.3-18.7
8.0	154.0	5.9	.74	19.0	16	3.3-18.7
8.5	168.0	6.4	.76	20.0	4	3.3-18.7
9.0	145.0	6.7	.74	16.0	23	3.3-18.7
9.5	145.0	7.0	.73	15.0	6	3.3-18.7
10.0	139.0	7.2	.72	14.0	6	3.3-18.7
11.0	167.0	7.8	.71	15.0	4	3.3-18.7
12.0	163.0	8.8	.73	14.0	2	3.3-18.7
13.0	165.0	8.5	.65	13.0	1	3.3-1 -7
Total	....	....	....	....	173	....
Mean, 6.68	131.8	4.86	.728	19.7	....	....

As is to be expected, equation (5), which is deduced for deep-water waves, almost invariably gives results too small when applied to waves in shallow water.

This equation, however, may be written as follows:

$$a = \frac{h}{2} + c' \frac{h^2}{L}, \qquad (5A);$$

in which c' is believed to be a constant for any particular locality at which observations are taken upon waves not near the point of breaking.

It was found that in the Duluth Ship Canal, with a mean depth of 26 feet, c' was equal to 2.0, and in Section I of the preceding table column 4 has been computed by equation (5A), giving to c' the value 2.0.

Column 6 is derived from columns 1 and 4.

All observations in Section II of the table were taken upon waves about to break in shoal water, and most of them were taken when strong winds were blowing in the direction of wave travel. The slope of the bottom varied from $\frac{1}{30}$ to $\frac{1}{90}$.

It will be noticed from column 4, Section II, that irrespective of the height or length of the wave, about 73 per cent of the wave height, $a=0.73h$, was above still-water level just before the wave broke. This result agrees well with that obtained from the observations at North Beach, Fla., $a=0.76h$, although most of the observations there were taken in calm weather and at a locality where the slope of the bottom was about $\frac{1}{100}$.

Most of the individual observations embraced in the preceding table were quite uniform, and differed but little from the mean values given, but in three instances unusual exceptions were noted.

The photograph, page 65, was taken May 24, 1901, in the Duluth Ship Canal, and shows a wave 9.2 feet in height and 160 feet in length. The lowest part of the wave hollow is 6 feet below the still-water level, while the highest point of the crest is but 3.2 feet above the same plane. This wave, therefore, partakes somewhat of the nature of a negative wave.

On May 1, 1902, a wave 8 feet in height and 150 feet in length was observed in the Duluth Ship Canal, the crest of which was 8.5 feet and the lowest point of the hollow 0.5 feet *above* still-water level.

On April 22, 1902, a wave 2.5 feet in height and 60 feet in length, which afterwards broke in a depth of 3.9 feet, was observed in Lake Superior in water of a mean depth of 4.7 feet, and it was noted that the crest was 2.7 feet, and the lowest point of the hollow 0.2 feet, *above* still-water level.

In the last two instances the waves were by definition true positive waves.

As three of these abnormal waves were noted out of 789 waves observed in all, it seems probable that during every continued storm, when many thousand waves reach shore, some of these abnormal waves would be found, provided observations were made during the entire period that the storm lasted.

Depth in which waves break.—Due to the action of high

winds, currents, and perhaps other causes not so well understood, oscillatory or deep-water waves *may* break, partially at least, in water of ample depth for their free propagation. Therefore, no matter how great the depth of the surrounding water, a barrier opposed to them *may* at times be subjected to the direct action of breaking waves.

On the other hand, it is well known that waves invariably break on reaching water of insufficient depth, and it is therefore of great importance to engineers that they should seek to discover the relation, if any, existing between certain dimensions of a wave and the minimum depth of water in which the wave can be propagated without breaking. For with this relation established, it will be possible, in many cases, to determine in advance the maximum wave which can assail a contemplated structure.

In observations heretofore taken for this purpose the wave height only has been considered, and this has been compared directly with the depth of the water in which the wave broke.

As would be anticipated, the results obtained at different localities do not show a precise ratio between the height of a wave and the depth of water in which it breaks.

Prof. G. B. Airy has shown mathematically that when the depth is *variable* it is impossible that a series of waves can exist having oscillatory motion of the particles and satisfying the equations of continuity and of equal pressure.

While the continuity holds equal pressure *will* exist; therefore in such a case continuity *must* cease, i. e., the water become broken.

This is, in his opinion, the explanation of the fact that the sea in places breaks over deeply submerged banks, shoals, or reefs, where the waves in general are high, as on the edge of the Banks of Newfoundland, where the depth is 500 feet, and about the line of "no soundings," that is, the line beyond which the water suddenly becomes deeper than 600 feet, which line borders the British Isles at some distance.

Observations upon ocean waves.—On the coasts of the United States the most violent wave action is probably found on the Pacific Ocean between San Francisco Harbor and the Straits of Fuca.

By act of Congress, approved March 3, 1879, the question of the selection of a site for a proposed harbor of refuge

on the Pacific Ocean between the Straits of Fuca and San Francisco, Cal., was referred to the Board of Engineers for the Pacific coast, and in pursuance of the duty intrusted to it, careful investigations were made by the Board as to the depth of water in which waves broke in the locality covered by the act mentioned. The following are extracts from the evidence upon this point collected by the Board:

Captain Maury, master of the *City of Tokio*, states: "I have seen the sea break in 8 or 10 fathoms."

Captain Debney states: "I have seen it myself breaking in 15 fathoms of water in a straight line on the coast. I lost a vessel on the coast that had decks stove in, moored in 7 fathoms. At Novarro River, in 1862 or 1863, I have known it to break in over 15 fathoms of water, straight along the coast. I have stood on Cape Disappointment and seen it do that. There was no guessing about it."

Capt. J. W. White, of the United States Revenue Marine Service, states: "I have seen it break in 15 fathoms off Cape Foulweather."

Capt. Frederick Bolles states: "I have seen it break in 10 fathoms off the Columbia, and in 8, 9, and perhaps 10 fathoms, off San Francisco bar."

Captain Connor states that between Trinidad Head and Pilot Rock he has seen it break in 7 or 8 fathoms, and thinks that it breaks in deeper water.

Prof. George Davidson, Assistant, United States Coast and Geodetic Survey, states that at Cape Mendocino the swell has been seen to break in 9¼ to 9½ fathoms of water, and he was of the opinion that he had seen waves breaking along the bars at the mouth of the Columbia River and at San Francisco bar in water of from 7 to 7½ fathoms in depth.

Mr. H. B. Tichenor states that at Port Orford he has seen waves break in a depth of from 10 to 11 fathoms; but, on the other hand, the testimony of an old resident of that place was that he had never seen a sea there break in over 8 fathoms.

The Board was of the opinion, after hearing the testimony, "that waves break into combers in depths of 8 and even 10 fathoms of water."

From letters furnished in April, 1902, through the courtesy of Capt. W. C. Langfitt, U. S. Corps of Engineers, the fol-

lowing additional data upon the same subject have been secured:

Mr. G. B. Hegardt, U. S. assistant engineer, states that in violent storms waves have been observed to break at, or a short distance outside, the whistling buoy over the Columbia River bar in a depth of 15 to 18 fathoms.

Captain Richardson, master of the U. S. Light House steamer *Manzanita*, in the fall of 1889 observed waves off the Washington coast, about 320 feet from hollow to hollow, break in 12 fathoms of water.

Mr. J. S. Polhemus, U. S. assistant engineer, states that in severe storms he has seen the waves near Yaquina and Coos bays, Oregon, break in a depth of 8 to 10 fathoms.

Off St. Augustine bar, Florida, in 1890–1891, the writer has seen waves breaking in water from 4 to 5 fathoms in depth during northeast storms which were not of exceptional severity.

In none of the preceding instances are sufficient data available to show any relation between the dimensions of the waves and the depth in which they break, but from the table of heights of deep-water waves, given in Chapter V, it is evident that in most if not all of the instances mentioned the depth in which the wave broke was much greater than any observed height of an ocean wave. From the character of the locality and from the degree of exposure, it seems reasonable to assume that the North Pacific coast of the United States is at times exposed to waves as great as any which have ever been observed. In fact, one of the largest recorded waves—that photographed from the U. S. S. *Albatross* and described in Chapter V—was encountered in the North Pacific at some distance off the mouth of the Columbia River. It will be seen, from what will subsequently be shown, that a wave over 50 feet in height would probably break in a depth greater than 14 fathoms and *might* break in a depth as great as 22 fathoms.

Observations by J. Scott Russell.—Mr. J. Scott Russell made a series of experiments upon positive waves in a channel of uniform depth, the sides of which were vertical and met in a vertical line, the plan of the channel being a triangle with one very acute angle, i. e., the channel was of uniform depth and gradually decreasing breadth.

He found that the wave always broke when its elevation above the general level became equal or nearly so to the general depth.

Observations were also made on the same class of waves in a channel of uniform breadth and gradually diminishing depth, the bottom of which was inclined at a slope of $\frac{1}{51}$, and it was found that in this case, as in the preceding one, the wave broke when its height above the general level was equal to the depth of the water at the spot.

The phenomena of waves breaking on the shore were observed principally on a very fine smooth beach of sand, having a slope toward the sea of 1 in 50; so perfectly plane and level was it, at the time when the observations were made, that a single wave a mile in breadth might be observed advancing to the shore so perfectly parallel to the edge of the water that the whole wave rose, became cusped, and broke at the same instant; a line of graduated rods was fixed in the water at different depths, from 6 inches to 6 feet in length, and it was observed that every wave broke exactly when its height above the antecedent hollow was equal to the depth of the water.

These particular results appear to conflict slightly with his general conclusions upon the subject, in which he states that he never noticed a wave so much as 10 feet high in 10 feet of water, nor so much as 20 feet high in 20 feet of water, nor 30 feet high in 5 fathoms of water; but he has seen waves approach very nearly to those limits.

Observations by Thomas Stevenson.—Mr. Thomas Stevenson, from observations made at the Firth of Forth on short, steep, and superficial waves, due to an existing wind, and breaking on a sandy beach, obtained the following results:

Total height of waves.	Depth of water below wave hollow.	Ratio of depth to wave height $=\frac{d}{h}$.
Feet.	*Feet.*	
2.5	1.16	0.71
3.0	1.42	.72
3.0	1.42	.72

In July, 1870, he made observations during a northeasterly ground swell at the seaward end of the iron pier at Scarborough. The following were the results for breaking waves:

Total height of wave.	Depth of water below wave hollow.	Ratio of depth to wave height $=\frac{d}{h}$.
Feet.	*Feet.*	
5.5	....	....
5.0	....	....
5.0	....	....
5.5	....	....
Mean = 5.25	10.25	2.20
6.0	....	....
6.0	....	....
8.0	....	....
6.0	....	....
6.0	....	....
Mean = 6.4	13.71	2.39

In each of the two preceding sets of observations by Mr. Stevenson, the third column has been added by the writer, under the assumption, based upon a large number of observations, that at the moment of breaking about three-fourths of the total wave height is above still-water level.

Mr. Stevenson states that some of the large waves in Wick Bay, during storms, were noticed to break when they came into water of the same depth as their height. He states that waves break in deeper water when the bottom shoals suddenly than when the slope is gradual.

Observations by Bazin.—In 1859, Henri Bazin inaugurated a series of experiments upon positive waves to determine the relation between the height of a wave and the depth of water in which it breaks.

He made use of a perfectly straight and regular channel about 6.5 feet wide, with the bottom inclined 1.5 feet in 1,000, giving him an opportunity to observe the effect of the uniformly diminishing depth upon the velocity and form of the wave. He found that positive waves generally broke when their height exceeded two-thirds of the total depth, a result not fully agreeing with Mr. J. Scott Russell's experiments, previously described, upon similar waves in an artificial channel.

Mr. William Shield states that "there is a shoal known as Riy Bank, off the south coast of Africa, where long ground swells, 10 to 12 feet in height, are commonly seen to break,

sometimes for days at a time, where the general low-water depth is 10 fathoms."

At Peterhead, in November, 1888, he measured waves 26 feet in height and 500 feet in length, which traveled with a velocity of 41 feet per second in water from 7 to 8 fathoms in depth, and crested and broke when passing the $5\frac{1}{2}$-fathom line.

From a staging in Algoa Bay, Mr. Shield measured unbroken passing waves 21 feet in height, in water 23 feet deep. These waves were formed by a wind velocity of 60 miles per hour. The bottom slope seaward was 4 fathoms per mile.

Observations at North Beach, St. Augustine, Fla.—Between January, 1890, and May, 1891, the writer made observations at North Beach, St. Augustine, Fla., upon 58 waves, varying in height from 4 inches to 6 feet just before breaking. Most of the observations were made during perfectly calm weather, the breakers being produced by very regular swells, resulting from disturbances having no connection with local conditions. The bottom was of fine sand, very smooth and hard, with a uniform slope of about $\frac{1}{100}$.

Denoting the depth, measured from the undisturbed water level by d, and the height of the wave immediately before breaking by h, and considering all observations taken, it was found that d varied between the limits $0.72\ h$ and $2\ h$, but for the greater number of observations d was equal to h.

For a given locality and a given slope, variations in the ratio of d to h appeared to be due almost entirely to the direction and force of the wind. With a strong wind blowing in the direction of wave travel, d became equal to $1.25\ h$, while with an equally strong wind blowing in the contrary direction d at the same locality was equal to $0.72\ h$. When there was no wind and the breakers were due solely to the ocean swell, the ratio of d to h varied with the slope of the bottom. For example, when the bottom was uniform and the slope about $\frac{1}{100}$, d became nearly equal to h, but when the slope was $\frac{1}{12}$, d became greater than $2\ h$.

As few, if any, engineering works give a less value for d than $d=h$, it may be well to state that there were in all 10 carefully taken observations in which d was found to be less than h, but in every case there was either no wind, or one directly opposed to the direction of wave travel. As has previously been shown, Mr. Thomas Stevenson's observations at the Firth of Forth gave $d=0.72h$.

Observations on Lake Superior.—At Eagle Harbor, Lake Superior, in 1875, waves from 10 to 12 feet in height were observed breaking in water of a depth about equal to their height.

In 1901–2 the writer instituted observations at four points on Lake Superior, for the purpose of ascertaining the ratio between the height of a wave and the depth of water in which it broke.

The results of these observations are given in the following table:

TABLE XVI.—*Depths in which waves broke, arranged according to wave heights.*

SEC. I. LAKE SUPERIOR, NEAR SOUTH PIER, DULUTH CANAL.

Wave.			Depth in which wave broke.	Ratio of depth to wave height $=\frac{d}{h}$.			Number of observations.
Height.	Length.	Velocity.		Maximum.	Minimum.	Mean.	
Feet.	*Feet.*	*f. s.*	*Feet.*				
2.0	....	....	4.0	2.00	2.00	2.00	3
2.5	....	14.0	3.9	....	....	1.56	1
3.0	70–75	14.6–18.2	4.3–5.8	1.93	1.43	1.64	6
3.5	63–75	12.2–15.0	4.5–7.4	2.11	1.29	1.54	5
3.7	68	10.4	6.3	....	....	1.70	1
4.0	70–82	16.0–16.6	5.3–7.4	1.85	1.32	1.48	4
4.5	90	16.0	5.3–10.7	2.38	1.18	1.78	2
5.0	....	18.0	11.2	....	....	2.24	1
5.5	144	24.1	14.9	....	....	2.71	1
5.7	....	....	10.7	....	....	1.88	1
6.0	94–164	16.9–21.0	9.3–15.9	2.65	1.55	2.22	6
6.5	109	21.1	14.2–15.7	2.41	2.18	2.29	2
6.8	150	18.0	14.4	....	....	2.12	1
7.0	94–167	15.4–24.1	10.7–17.0	2.43	1.53	2.10	8
7.2	....	. . 21.0	10.7–15.7	2.18	1.49	1.95	3
7.5	....	18.0	12.3	....	....	1.64	1
7.8	170	15.5	14.0	....	....	1.80	1
8.0	163–180	16.9–24.0	13.2–20.9	2.61	1.65	2.07	9
8.5	170	18.0 .	13.2–14.0	1.65	1.55	1.60	2
8.7	....	18.0	12.3	....	....	1.41	1
9.0	114–180	16.9–24.0	12.6–20.9	2.32	1.40	1.81	17
9.5	89–160	15.4–21.0	15.2–19.9	2.09	1.60	1.80	4
10.0	99–159	18.8–21.1	14.3–14.7	1.47	1.43	1.45	4
11.0	160–169	19.5–26.0	12.8–20.0	1.82	1.16	1.59	4
13.0	....	24.1	17.9	....	....	1.38	1
Total..	....	....	....	....	....	....	89
Mean..	....	....	....	2.13	1.52	1.84	....

TABLE XVI.—*Depths in which waves broke, arranged according to wave heights*—Continued.

SEC. II. LAKE SUPERIOR, NORTH OF NORTH PIER, DULUTH CANAL.

Wave. Height.	Wave. Length.	Wave. Velocity.	Depth in which wave broke.	Ratio of depth to wave height $=\frac{d}{h}$. Maximum.	Ratio of depth to wave height $=\frac{d}{h}$. Minimum.	Ratio of depth to wave height $=\frac{d}{h}$. Mean.	Number of observations.
Feet.	*Feet.*	*f. s.*	*Feet.*				
2.0	45	9.1	2.6			1.30	4
2.5	55–70	10.5–13.3	2.5–3.5	1.40	1.00	1.18	7
2.6	65	11.2	2.7			1.03	4
2.8	70	12.5	4.1			1.46	1
3.0	50–75	12.5–13.3	4.7–5.0	1.67	1.57	1.62	7
3.3	55	14.3	4.4			1.33	1
3.5	60	12.5	4.2–4.6	1.31	1.20	1.26	2
6.0	75		11.0			1.83	2
6.5	80		11.0			1.70	1
Total							29
Mean						1.36	

SEC. III. LAKE SUPERIOR, NEAR PRESQUE ISLE POINT, MICHIGAN.

Height.	Length.	Velocity.	Depth in which wave broke.	Maximum.	Minimum.	Mean.	Number of observations.
6.0	100		7.4			1.23	3
9.0	75–80		12.0–12.4	1.38	1.33	1.36	6
Total							9
Mean						1.32	

SEC. IV. LAKE SUPERIOR, NEAR GRAND MARAIS, MICHIGAN.

Height.	Length.	Velocity.	Depth in which wave broke.	Maximum.	Minimum.	Mean.	Number of observations.
7.5	120	20.0	10.0			1.33	3
8.0	95	20.0	10.0			1.25	4
Total							7
Mean						1.28	

Mean value for $\frac{d}{h}$ for 134 observations in all=1.67.

NOTE.—In Section I of the preceding table the general slope of the bottom was as follows:

Between depths of 2 feet and 8 feet, $\frac{1}{50}$.
Between depths of 8 feet and 12 feet, $\frac{1}{30}$.
Between depths of 12 feet and 16 feet, $\frac{1}{40}$.
Between depths of 16 feet and 21 feet, $\frac{1}{30}$.
In Section II, the bottom slope was $\frac{1}{40}$; in Section III, $\frac{1}{70}$, and in Section IV, $\frac{1}{85}$.

The slope of the bottom was less uniform for the observations in Section I than for those in the other sections.

The average wind velocity during the observations in Section I was 30.4 miles per hour. For 10 observations in Section II, the velocity of the wind was 26.7 miles per hour, and for the other 19 observations there was either no wind, or but a gentle breeze. The ratio $\frac{d}{h}$ in the two cases was 1.42, and 1.34, respectively.

For the observations in Section III, the wind velocity is not known, and for those in Section IV it was estimated at 25 miles per hour. The direction of the wind was always the same as that of wave travel.

FREE WAVE 5 FEET IN HEIGHT, BREAKING IN WATER 8.9 FEET IN DEPTH, LAKE SUPERIOR, 1902.

It will be noticed from an inspection of the preceding table that the ratio $\frac{d}{h}$ varies from 1 to 2.71, the average value for the 134 observations being 1.67.

It was found that this ratio *decreased* with a decrease in wind velocity and in slope of the bottom, and *increased* with an increase of wave length and of irregularity of the bottom.

It appears probable that the undertow also plays an important part in causing waves, apparently similar in other respects, to break in different depths of water. The undertow rushes out from shore with very variable velocity, and it is fair to suppose that if the front of a wave encounters a strong opposing undertow, the wave will break sooner than if but a feeble undertow, or none at all were encountered.

It will be noticed from an inspection of the photograph on page 123 how broad and flat is the hollow, and how steep and narrow is the crest, in the case of a breaking wave. This photograph was taken during a calm on the day following a storm. The waves were very regular, and the one shown, which was about 5 feet in height, broke in a depth of about 8.9 feet. The bottom was sandy and uniform, the slope being about $\frac{1}{37.5}$.

This may be taken as a typical breaking wave uninfluenced by wind, and shows how widely such a wave departs from the form of the common cycloid at the hollow, although resembling it fairly closely at the crest.

If, as is sometimes contended, the wave-form at the instant of breaking is that of the common cycloid, the wave length in this case would be but 3.14 times the wave height, whereas it is about 21 times the wave height, and in Section II of Table XV, for the mean of 173 observations, it is 19.7 times the wave height.

Photographs of unbroken waves in the outer end of the Duluth Ship Canal, taken about half an hour before that of the breaking wave just described, are shown on pages 62, 63, and 64.

CHAPTER VIII.

FORCE EXERTED BY WAVES.

How wave energy is exerted and transmitted. Order in which the subject of wave force is treated. Instances of force exerted by ocean waves and by waves on the Great Lakes.

We have seen that the total energy of a deep-water wave, during a complete wave period, is $E=\frac{wLh^2}{8}\left(1-4.935\frac{h^2}{L^2}\right)$, of which one-half is kinetic and one-half potential, the latter half being transmitted onward with the wave form. The relative theoretical distribution of wave energy between the line of centers of service orbits and any depth d′ is given by equations (9a) and (9b), Chapter IX, and is shown graphically for particular cases in Pl. V.

If wave motion is arrested by any interposing barrier, a part, at least, of the energy of the wave will be exerted against the barrier itself, and unless the latter is strong enough to resist the successive attacks of the waves, its destruction will ensue.

No other force of equal intensity so severely tries every part of the structure against which it is exerted, and so unerringly detects each weak place or faulty detail of construction.

The reason for this is found in the diversity of ways in which the wave force may be exerted and transmitted; for example: (1) The force may be a static pressure due to the head of a column of water; or (2) it may result from the kinetic effect of rapidly moving particles of the fluid; or (3) to the impact of a body floating upon the surface of the water and hurled by the wave against the structure; or (4) the rapid subsidence of the mass of water thrown against a structure may produce a partial vacuum, causing sudden pressures to be exerted from within.

These effects may be transmitted through joints or cracks in the structure itself; (*a*) by hydraulic pressure; or (*b*) pneu-

matic pressure; or by a combination of the two; or (*c*) the shocks or vibrations produced by the impact of the waves may be transmitted by means of the materials of which the structure itself is composed.

In the discussion of wave force which follows the subject will be considered in the following order:

1°. Instances of force exerted by ocean waves and by waves on the Great Lakes.

2°. Depth to which wave action extends.

3°. Measurements of the force of waves by means of dynamometers provided with springs.

4°. Measurements by means of hydrostatic dynamometers having diaphragm disks.

5°. Experiments made to ascertain the character of the force exerted by waves and to determine whether this force can properly be measured by dynamometers in units of pressure.

6°. Comparison of theoretical with observed wave force.

INSTANCES OF FORCE EXERTED BY OCEAN WAVES AS DESCRIBED BY VARIOUS AUTHORITIES.

(1) The Carr Rock beacon was overturned in November, 1718, soon after being finished, by the waves of the German Ocean. This beacon was a column of freestone 36 feet in height and 17 feet at the base. The diameter at the plane of fracture was 12 feet 9 inches, and at the level of high water 11 feet 6 inches.

(2) During the construction of the Dhuheartach light-house, completed in 1872, fourteen stones of 2 tons weight each, which had been fixed into the tower by joggles and Portland cement at the level of 37 feet above high water, were torn out and carried into deep water.

(3) At Wick, in February, 1872, during the construction of the harbor works, and after the superstructure had been rebuilt solidly with Portland cement, the face stones in many places were shattered by the sea, a phenomenon without parallel in the history of works of this class. The stones were of the same density as granite, and were stated to have been three times stronger than Craigleith stone.

(4) On February 15, 1853, during a northeast gale, a large body of water was thrown upon the lantern of Noss Head

light-house, Caithnessshire, which was situated about 175 feet above the sea.

(5) At Ymuiden, the harbor entrance of the Amsterdam Canal, the breakwaters are vertical, or nearly vertical, structures, with mounds of concrete blocks on the seaward side. During a gale a 20-ton block was lifted by a wave vertically to a height of 12 feet and landed upon the top of the pier, which was 4 feet 10 inches above high water.

During another gale at the same place a "header" block in the seaward face of the pier, 7 by 4 by 3.5 feet, was started *forward* out of its place by the stroke of a wave compressing the air in rear of it. This block weighed about 7 tons, and the top was at low-water level. This layer was built dry, but was surmounted by three courses of concrete blocks, each 3 feet thick, set in Portland cement mortar.

(6) At Cherbourg the breakwater is composed of an immense embankment of loose stone protected in places by concrete blocks measuring 700 cubic feet each. The embankment of stone is surmounted by a wall 20 feet in height. During a very severe storm December 25, 1836, stones weighing nearly 7,000 pounds were thrown over the top of the wall, and many of the huge concrete blocks were moved—some of them as far as 60 feet—and two of them were overturned.

(7) Hagen states that on August 20, 1857, during a storm in the harbor of Cette, a block of concrete, 2,500 cubic feet in volume and weighing about 125 tons, was moved upon its bed for a distance of more than 3 feet.

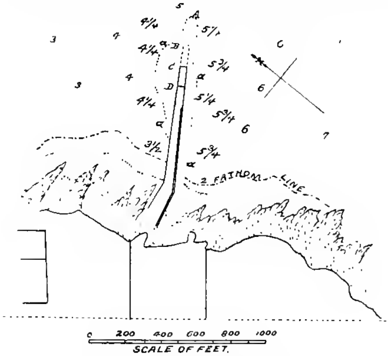
(8) At Unst, the most northerly of the Zetland Islands, a door was broken open at a height of 195 feet above the sea.

(9) At Peterhead Harbor, in January, 1849, three successive waves carried away 315 feet of a bulwark 9½ feet above high water of spring tides, one piece, weighing 13 tons, being moved a distance of 50 feet.

(10) At Bound Skerry, Shetland group of islands, a block of stone, 5.5 tons in weight and situated 72 feet above the level of high water-spring tides, was detached from its bed and moved over 20 feet. Another block, weighing nearly 8 tons, had been torn up and driven before the waves over several ledges with nearly vertical faces of from 2 to 7 feet in height for a distance of 73 feet at a general level of 20 feet above the high water of spring tides.

WICK BAY.

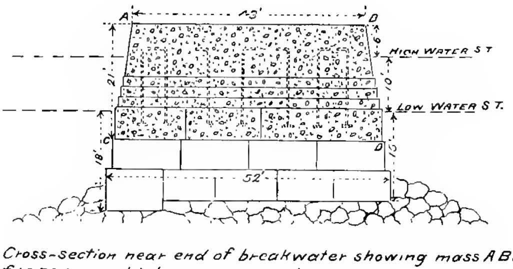
NOTE. Depths are in fathoms at Low Water Spring Tides.



REFERENCES.

A. Extreme distance to which the Rubble base was carried.
B. " " " " " superstructure was carried.
C. Point to which the building was restored & finished by Cement Blocks.
D. End of building left standing.

The dotted line a shows the extent of the Bay occupied by Rubble laid down from the staging or spread by storms as last ascertained after the damage of Feb. 1870.



Cross-section near end of breakwater showing mass ABCD of 1350 tons which was removed entire.

FROM CONSTRUCTION OF HARBORS, BY THOS STEVENSON.

(11) The damage to Wick breakwater in 1872 is described as follows by the Messrs. Stevenson:

The end of the work, as has been explained, was protected by a mass of cement rubble work. It was composed of three courses of large blocks of 80 to 100 tons, which were deposited as a foundation in a trench made in the rubble. Above this foundation there were three courses of large stones carefully set in cement, and the whole was surmounted by a large monolith of cement rubble, measuring about 26 by 45 feet by 11 feet in thickness, weighing upward of 800 tons. The block was built *in situ.* As a further precaution, iron rods 3.5 inches in diameter were fixed in the uppermost of the foundation courses of cement rubble. These rods were carried through the courses of stonework by holes cut in the stoue, and were finally embedded in the monolithic mass, which formed the upper portion of the pier. (See Pl. IV.)

Incredible as it may seem, this huge mass succumbed to the force of the waves, and Mr. McDonald, the resident engineer, actually saw it from the adjacent cliff being gradually "slewed" round by successive strokes until it was finally removed and deposited inside the pier. It was not for some days after that any examination could be made of this singular phenomenon, but the result of the examination only gave rise to increased amazement at the feat which the waves had actually achieved. It was found on examination by diving that the 800-ton monolith forming the upper portion of the pier, which the resident engineer had seen in the act of being washed away, had carried with it the whole of the lower courses, which were attached to it by the iron bolts, and that this enormous mass, weighing not less than 1,350 tons, had been removed *en masse* and was resting entire on the rubble at the side of the pier, having sustained no damage but a slight fracture at the edges. A further examination also disclosed the fact that the lower or foundation course of 80-ton blocks, which were laid on the rubble, retained its position unmoved. The second course of cement blocks, on which the 1,350-ton mass rested, had been swept off after being relieved of the superincumbent weight, and some of the blocks were found entire near the head of the breakwater. The removal of this protection left the end of the work open, and the storm, which continued to rage for some days after the destruction of the cement rubble defense, carried away about 150 feet of the masonry (one-seventh of the whole), which had been built solid and set in cement. The same remarkable feature of former damage was strikingly apparent in the last damage,—*the foundations, even to the outer extremity of the work, remaining uninjured.*

(12) Remarkable as is this example, it was exceeded in 1877, when another mass of concrete substituted for that which had been carried away in 1872, was itself carried away, although containing about 1,500 cubic yards of cement rubble and weighing about 2,600 tons.

(13) The United States assistant engineer at Coos Bay, Oregon, states that blocks of stone weighing over 10 tons have been washed off the jetty enrockment above high tide

by storm waves. The outer end of the jetty for a distance of 600 or 700 feet has twice been built up to a height of over 20 feet above low tide with blocks of stone weighing from 2 to 10 tons each, but in 1902 this portion of the jetty had been beaten down by waves so that the crest of the enrockment was about 10 feet below low tide level.

The stone used in the jetty at Coos Bay was gray sandstone, weighing about 147 pounds per cubic foot.

(14) Tillamook Rock, on the coast of Oregon, 18 miles south of the mouth of the Columbia River, upon which is located an important light-house, is exposed to tremendous wave action.

During every severe storm fragments of the rock are torn off and thrown on the roof of the keeper's dwelling.

In December, 1894, one fragment weighing 135 pounds was thrown clear above this building (by one of the United States assistant engineers it is stated to have gone higher than the light), and in falling broke a hole 20 feet square through the roof, practically wrecking the interior of the building. Thirteen panes of glass in the lantern were broken during the same storm. The base of the keeper's dwelling is 91 feet above low water, and the focal plane of the light is 139 feet above the same datum. The mean range of high tides above the lower low waters is a little more than 7 feet.

(15) On another occasion a fragment of rock, weighing about half a ton, was rolled across the platform at the base of the building, 91 feet above low water, wrecking a wrought-iron fence.

(16) The keeper of the same light reports that on February 11, 1902, spray or waves were thrown to a height of fully 100 feet above the roof of his dwelling, or more than 200 feet above the level of the sea, "descending in apparently solid water on the roof."

(17) At North Beach, St. Augustine Harbor, Florida, in 1890, a block of concrete 2.5 by 6 by 10 feet, weighing 10.5 tons was lifted vertically 3 inches and there caught and held fast by a piece of stone. The depth of water in the immediate vicinity did not exceed 6 feet at highest tides, and the largest observed breakers at such times never exceeded 5.5 feet in height. A dynamometer located within a few feet registered a maximum pressure of 633 pounds per square foot on this occasion.

SOUTH HARBOR BREAKWATER, BUFFALO, N. Y.

Showing damage by storm of September 12, 1900. Broken ties spaced 5 feet between centers. No longitudinals.

INSTANCES OF FORCE EXERTED BY WAVES ON THE GREAT LAKES.

(18) During a severe storm, January 1, 1891, at the harbor of refuge at Milwaukee, Wis., a rock-filled timber crib 100 feet long, 24 feet wide, and 22½ feet high was overturned by waves in water of a depth of 32 feet. The crib rested upon a mound of riprap 12 feet in height, with exterior slope of 1 on 2, and interior slope of 1 on 1½, and top width of 40 feet, leaving an 8-foot berm on each side of the crib. During this storm the wind, which was from the northeast, attained a maximum velocity of 48 miles per hour.

(19) At the same locality, during a storm, April 19, 1893, two adjoining cribs, each 50 feet long, 24 feet wide, and 26½ feet high, were overturned by wave action. These cribs were in water 29 feet in depth, and rested upon a riprap mound 9 feet in height, and with the same side slopes as in the previous case. The berm on the lake side was 10 feet and that on the harbor side 6 feet.

(20) During an especially severe gale at Buffalo, N. Y., December 12, 1899, considerable damage was inflicted by waves upon the parapet and upper deck of sections one and two of the south harbor section of the timber crib breakwater, which sections had been completed but a few weeks previously. Seventy 12 by 12 inch by 12 feet timber ties, 10 feet between supports and spaced 5 feet from center to center, were broken in the middle by the impact of the falling water, which was thrown to a great height upon striking the vertical lake face of the breakwater. These timber ties supported 3-inch longitudinal plank decking placed at right angles to the ties.

The superstructure of the timber crib breakwater was constructed in three benches, the first 6 feet high and 12 feet wide, the second 4 feet high and 12 feet wide, and the third, or parapet, 2 feet high and 12 feet wide, making the height on the lake side 12 feet above mean lake level.

(21) The same breakwater, which was finally completed October 27, 1900, suffered great damage from the gale of September 12, 1900 (see photograph, p. 129), and still greater damage from that of November 21, 1900, described as follows by the officer in charge, Maj. T. W. Symons, U. S. Corps of Engineers: "This gale was of unusual violence, the wind blowing at times at the rate of 80 miles per hour from

the west." The water level of the lake varied from 3 feet below mean lake level to 6.4 feet above the same datum between 4 p. m. and midnight.

During this gale "tremendous seas broke over the breakwater. The waves, dashing against the vertical walls of the structure, rose to a great height above it, variously estimated at from 75 to 125 feet, enveloping the breakwater in an immense sheet of water, which in falling struck the top of the superstructure with such force as to crush in the same, the large timbers of which it was constructed being broken like pipestems. The direction of the breakwater being at right angles to the axis of the storm tended further to accentuate the destroying power of the furious waves."

About 900 feet of superstructure in all was razed almost to the water's edge.

"On section 4 the entire middle deck was razed to the banquette level. As on section 3, the parapet withstood to some extent the effect of the gale, the ties having been spaced 3 feet 4 inches centers, instead of 5 feet, as usual."

(22) At the same harbor, a section of the concrete banquette of the superstructure of the breakwater weighing about 208 tons was displaced during a heavy storm, one end being shifted bodily more than 2 feet.

This section was 36 feet in length and 14 feet in width (see photograph, p. 130). Its upper surface was about 4.5 feet above mean lake level. The concrete of which it was composed weighed 151 pounds per cubic foot.

(23) At the north breakwater, Buffalo Harbor, New York, during the storm of September 12, 1900, when the wind attained a velocity of 78 miles per hour, and the lake level fluctuated from 2.6 feet below mean level to 5.7 feet above the same datum, a number of the lake face concrete blocks were washed overboard, falling lakeward.

These blocks contained 9.45 cubic yards of concrete each, and weighed 18.9 tons. They extended from 2 feet below to 3 feet above mean lake level, the portion of the lake face of the block above mean lake level presenting a sloping surface to the waves, the angle of slope being 34° from the vertical. The base of each block was 7.2 by 8 feet and the height 5 feet. The rear face was vertical, and its area 36 square feet. On the abutting ends the blocks were provided with joggle recesses,

CONCRETE SUPERSTRUCTURE OF BREAKWATER, BUFFALO, N. Y.

which were filled with concrete after the blocks were placed, forming a dowel to hold the blocks together and prevent water from washing through the joint.

(24) During a storm in the fall of 1899 at the same harbor four manhole covers were lifted from their places on the parapet deck of the concrete-shell type of breakwater, and two were washed overboard and two deposited on the banquette deck. The manhole covers were circular plates of concrete 3 feet in diameter and 6 inches thick, weighing about 530 pounds, with upper surface about 12 feet above mean lake level.

The officer in charge states that the movement of these covers was evidently caused by the high air pressure under the concrete shell, due to wave pulsation.

During a gale on the night of September 11, 1900, when the wind attained a velocity of 78 miles per hour, several manhole covers were again displaced. On this occasion the water rose 5.6 feet above mean lake level.

(25) One of the most striking instances of the force exerted by waves on the Great Lakes was observed by the writer at Black Rock, Presque Isle Park, Marquette, Mich., in June, 1902.

Black Rock is a large flat rock sloping toward and into Lake Superior, with an average slope of about $\frac{1}{10}$. The surface of the rock has been worn smooth and fairly regular by the tremendous force of the waves during northeast storms, to the action of which this locality is fully exposed. Near the highest part of the rock and at some distance from the water, three large masses of stone, with faces totally unlike the worn surface upon which they rested, were observed. Closer inspection showed that one of these stones had recently been moved several feet inland from a position which it had long occupied; which position was clearly marked on the black surface of the rock by a much lighter area, the outline of which conformed exactly to the bottom face of the nearest mass of stone. No one on seeing it could doubt the agency which had caused its removal, but if further proof were needed it was afforded by a photograph of a huge wave taken at the same point five years previously, in which two large masses of stone were shown in the front portion of the wave,

neither of which are to-day within 75 feet of the positions they then occupied.

At a distance of about 165 feet from the water line of the lake the rock slopes to the rear and is covered by soil and trees. Against the latter are piled numerous masses of stone which have been carried by the waves over the summit of the rock and rolled down the reverse slope.

The dimensions of the three masses of stone previously described, their distances from the water line of the lake, and the present heights of their centers of gravity above low-water datum are as follows:

Stone.	Dimensions.	Weight (tons of 2,000 pounds).	Distance from lake.	Height of center of gravity above low-water datum.	Remarks.
	Feet.		*Feet.*	*Feet.*	
(a)	5 by 3 by 1.5 ..	1.89	85	12	Sharp edges and corners.
(b)	6 by 3.5 by 3 ..	5.29	155	16	Rounded corners and edges.
(c)	6 by 6 by 2....	6.05	125	15	Sharp edges and corners; recently moved.

When the weight of the stones, the distance from the lake, and the height above it to which they were carried are considered, the tremendous force of the wave, before so much of its energy was dissipated in overcoming height and distance, can be realized.

(26) At Passage Island, in the northern part of Lake Superior, the light keeper's dwelling is situated upon a narrow, rocky island, the entrance to the house being 140 feet from the water and 60½ feet above it. The slope between the house and water is rough and irregular. During a northeast storm about the year 1897 the keeper states that "a solid mass of blue water about 2 feet deep" swept past the entrance to his dwelling, carrying away a board walk leading to it, and doing other damage of a minor character.

(27) The light-house at Stannard Rock, in Lake Superior, consists of a tower built upon an isolated, submerged rock, south of the middle of the lake, and exposed to the full force of the waves.

The light keeper states that in severe storms a solid mass of water rushes over the deck of the cylindrical foundation pier,

which is 23 feet above the water and 62 feet in diameter, and heavy spray is driven to a height of more than 102 feet above lake level.

(28) At Marquette Harbor, Michigan, during a storm June 28, 1899, thirty concrete blocks, 7 by 5 by 2 feet, with sloping lake face, containing 2.22 cubic yards each, and weighing 4.52 tons (dry) were washed overboard from the breakwater, some of them being carried to a horizontal distance of 60 feet. These blocks, which were 2 feet in height, rested on a horizontal bed, and their upper surfaces were just awash.

(29) During an unusually severe storm on Lake Superior September 16, 1901, a number of the 6 by 12 inches by 16 feet white-pine planking, covering the sloping lake face of the west breakwater at the upper entrance of the Portage Canal, were pulled loose from the timbers to which they were spiked and were broken in two about the middle. The sloping face of this breakwater extended from 4.25 feet below lake level (at the time) to 6.75 feet above the same datum. A dynamometer located within a few feet of the spot, and at an elevation of 7.5 feet above the water, gave a maximum reading of 2,525 pounds per square foot. The depth of the water was 30 feet, the velocity of the wind about 41 miles per hour, and the height of the crests of the highest waves above undisturbed water level 11.8 feet.

(30) At Marquette, Mich., during a storm in December, 1895, the 4 by 12 inch wooden decking of the breakwater, as well as the 12 by 12 inch wooden ties which supported it, were broken in several places. The ties were spaced 9 feet from center to center, and rested upon 12 by 12 inch supports, which were themselves 9 feet between centers.

(31) In 1899, six concrete footing blocks had been placed in position on the pierhead of the south pier of the Duluth Ship Canal. These blocks contained 3.39 cubic yards of concrete each, and weighed (dry) 6.88 tons. The area of the face presented to the waves was 18.77 square feet, and the bottom of each block rested in a 6-inch depression in the decking of the timber crib beneath it. Their upper surfaces were just awash.

During the storm of October 12–15, 1899, five of these blocks were lifted out of the depression in which they rested and washed overboard, and one was carried along on the top

of the substructure of the pier by waves for a distance of about 400 feet and lodged against a pile of footing blocks, where it was found turned bottom upward.

(32) At Duluth, Minn., during an unusually severe storm in September, 1881, a mass of trap rock about 2 cubic yards in volume, and weighing about 4.5 tons, was raised by a wave from its position alongside the vertical face of the old breakwater in Lake Superior and cast upon the deck of the breakwater, where it remained for many months. The depth in the vicinity was about 14 feet, and the mass of stone was raised vertically between 5 and 6 feet, the original position having been at about still-water surface level.

(33) Four cast-iron lamp-posts in all have been broken off by waves on three different occasions in 1901–2, from the pierheads at the Duluth Ship Canal, the point of fracture in each case being about 19 feet above the undisturbed level of the lake. The maximum observed wave heights in the same locality, on these occasions, were 20, 16, and 13.5 feet, respectively. The damage was caused by water, which was deflected upward and to one side on striking the pointed extremity of the pier. Four other lamp-posts, located upon the top of the parapet walls of the concrete piers, have been broken off by waves higher than the top of the parapet wall and traveling parallel to the pier, the point of fracture being about 11.8 feet above still-water level. The parapet wall is nearly vertical, and is 3 feet in width at the top. The lamp-posts were equidistant from the two faces of the wall.

CHAPTER IX.

Depth to which wave action extends. How determined theoretically. Instances in the case of ocean waves, and of waves on the Great Lakes.

In the case of deep-water waves the dimensions of the orbits of the particles is given by equation (7), and is shown graphically in Pl. V.

The total energy, both kinetic and potential, due to the motion of all particles between the surface and the depth corresponding to any orbit radius r is—

$$E'_{K+P}=\frac{2w\pi}{R}\left[\frac{r^4-r_s^{\,4}}{4}-\frac{R^2}{2}(r^2-r_s^{\,2})\right]; \qquad (9a).$$

From the preceding equations the theoretical motion of the particles and the total energy between any two depths can be computed provided the wave height, length and velocity are known.

For shallow-water waves—

$$E'_{K+P}=\frac{wL}{2}(b_s^{\,2}-b'^2)-\frac{9.87w}{L}(a_s^{\,2}b_s^{\,2}-a'^2b'^2); \qquad (9b).$$

DISTRIBUTION OF WAVE ENERGY.

The relative theoretical distribution of energy below the line of surface-orbit-centers is shown graphically to the right of the line AB in Pl. V, for both deep and shallow water waves in which $\frac{L}{h}=15$ (a very common ratio between height and wave length), and $\frac{d}{L}=0.1$ in the case of the shallow-water wave. To the left of AB is shown the relative dimensions of the orbit radii and semi axes for both waves at various depths. The scale for relative energy is the same for both waves, as is also that for orbit dimensions.

An inspection of the figure will show that although in this case the total theoretical energy of the shallow-water wave is

only about 95 per cent of that of the deep-water wave of equal height and wave length, yet in the case of the shallow-water wave an amount of energy equal to more than half the total energy of the entire deep-water wave lies above a horizontal plane situated at a distance 0.03L below the line of surface-orbit centers, while in the case of the deep-water wave only about 31 per cent of the total energy of the wave lies above the same plane.

It should be remembered that for a shallow-water wave like that described about three-fourths of the wave height lies above still-water level, consequently the line of surface-orbit centers is usually at a height about $=\frac{h}{4}=\frac{L}{60}$, above still-water level.

For such a wave 10 feet in height and 150 feet in length in water 12.5 feet in depth (a depth about as small as this wave could ordinarily exist in without breaking) over 53 per cent of the total theoretical energy of the wave lies above a horizontal plane only 2 feet below the surface of the water.

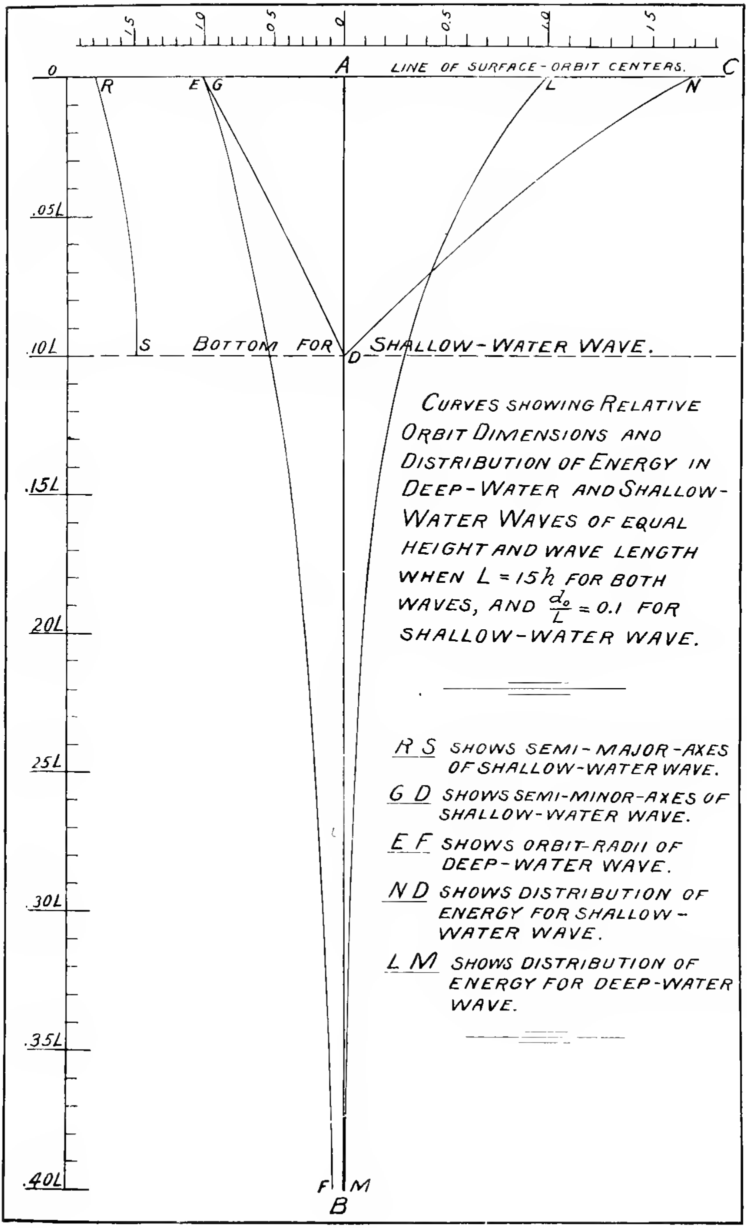
Pl. V shows graphically why the destructive effects of shallow-water waves are so great, and, taken with what has just been explained, shows why the maximum wave force is usually exerted at a considerable height above still-water level, and why it decreases so rapidly below the surface as the depth increases.

As shown by the dynamometer readings at the outer end of the south pier of the Duluth Canal, the maximum force for the larger storm waves (see fig. 13) was recorded at an average height of about 4½ feet above the water surface.

When a wave encounters an obstacle there may be developed at some particular depth below the surface an amount of energy in excess of that indicated by theory for the depth in question, the result being due to concentration or reflection of energy from other parts of the wave, or to "back draft," caused by the obstruction itself. This is especially true in the case of breakwaters with vertical faces, constructed upon rubble mounds.

In some instances in breakwaters of this type rubble of given dimensions has been displaced from along the vertical faces to a depth of 20 to 25 feet, when previous to the construction of the portion of the breakwater surmounting the

PLATE V


1.5
1.0
0.5
0
0.5
1.0
1.5
0
A
LINE OF SURFACE-ORBIT CENTERS.
C
R
E
G
L
N
.05L
.10L
S
BOTTOM FOR
D
SHALLOW-WATER WAVE.
.15L
.20L
.25L
.30L
.35L
.40L
F
M
B
CURVES SHOWING RELATIVE ORBIT DIMENSIONS AND DISTRIBUTION OF ENERGY IN DEEP-WATER AND SHALLOW-WATER WAVES OF EQUAL HEIGHT AND WAVE LENGTH WHEN $L = 15h$ FOR BOTH WAVES, AND $\frac{d_0}{L} = 0.1$ FOR SHALLOW-WATER WAVE.
RS SHOWS SEMI-MAJOR-AXES OF SHALLOW-WATER WAVE.
GD SHOWS SEMI-MINOR-AXES OF SHALLOW-WATER WAVE.
EF SHOWS ORBIT-RADII OF DEEP-WATER WAVE.
ND SHOWS DISTRIBUTION OF ENERGY FOR SHALLOW-WATER WAVE.
LM SHOWS DISTRIBUTION OF ENERGY FOR DEEP-WATER WAVE.

rubble mound it had remained undisturbed within 12 or 15 feet of the water surface.

This effect is so generally recognized that in most cases where a masonry superstructure surmounts a rubble mound it is customary to cover the adjacent exposed side slope of the mound with an apron or revetment of extra-heavy masses of concrete or rubble for its better protection.

In order that the effects of wave action at various depths below the surface of the water may be better appreciated, it is advisable at this point to give a few instances of these effects both in the case of ocean waves and of waves on the Great Lakes.

INSTANCES OF WAVE ACTION AT VARIOUS DEPTHS.—OCEAN WAVES.

(*a*) During the great storm at Wick Breakwater, in December, 1872, which has been described in paragraph (11) of the preceding chapter, the entire course of concrete blocks below CD, Pl. IV, were swept off by the sea, after being relieved of the weight of the superincumbent mass. These blocks weighed from 80 to 100 tons each, and had exposed faces of about 105 square feet. Their upper surfaces were 5 feet below low-water spring tides, and their lower surfaces 10 feet below the same datum. The range of spring tides is 10 feet, and the depth in the vicinity, referred to low-water spring tides, is about 30 feet.

Under the course of displaced blocks was a foundation course of similar blocks, weighing about 80 tons each and with exposed faces of 6 by 10.4 feet. The upper surfaces of these blocks were 10 feet below low-water spring tides, and their lower surfaces 16 feet below the same datum. The blocks of the entire foundation course, even to the outer extremity of the work, retained their positions unmoved during the storm.

At the breakwater at Peterhead, where waves 30 feet in height and from 500 to 600 feet in length are occasionally encountered, blocks weighing over 40 tons each have been displaced at levels below low-water ranging from 17 to 36 feet, and water thrown upward to a height of 120 feet.

(*b*) At Coos Bay, Oregon, the outer end of the United States jetty, for a distance of 600 or 700 feet has twice been

built up to a height of over 20 feet above low tide, with blocks of grey sandstone of from 2 to 10 tons each, of a density of about 147 pounds per cubic foot; but in 1902 this portion of the jetty had been so beaten down by waves that the crest of the enrockment was then about 10 feet below low tide level.

(*c*) Sir J. Coode found, from examination made in a diving suit, that the shingle of Chesil Bank was moved during heavy winter storms at a depth of 8 fathoms. Capt. E. K. Calver, R. N., states that waves 6 or 8 feet in height have been seen to change their color from stirring up the bottom on passing into water 7 or 8 fathoms in depth.

(*d*) Mr. Thomas Stevenson states that drift stones upward of 30 cubic feet in volume, and weighing more than 2 tons, have been thrown from deep water during storms upon the submerged rock on which the Bell Rock light-house is located.

(*e*) Sir John Robinson states that at Madras, during a violent storm, a quantity of pig lead was cast on the beach, which proved to have come from a vessel wrecked more than a mile offshore.

(*f*) Mr. John Murray on one occasion, when working in a diving bell in 20 feet of water, at Sunderland, found it impossible to work on account of the great agitation at the bottom, although at the surface there was but a slight swell, showing a movement of translation on the bottom, while at the surface there appeared to be only a movement of oscillition.

He also mentions that shingle and chalk ballast thrown overboard from ships off Sunderland at a distance of 7 to 10 miles from shore, in water of at least 10 fathoms in depth, were brought ashore in large quantities during violent storms and cast upon the beach by wave action.

(*g*) Mr. Kiddle notes that pilots and masters of vessels assert that off Nantucket Shoals sand is frequently left on deck by a sea which has broken on board, although the depth there is from 75 to 90 feet.

(*h*) Mr. Thomas Stevenson has called attention to the fact that in many localities one of the best indications of the depth to which appreciable wave action extends is afforded by the depth at which mud is found to repose upon the bottom. This substance is so easily moved by water in motion that when it is found upon the bottom of a body of water its pres-

ence clearly indicates lack of sufficient wave action to stir it up and carry it off.

The entire absence of mud at a place does not afford proof that there is appreciable wave action in that locality, for the geological conditions may prevent it ever having been formed, or if formed, it may have been carried away by currents.

Mr. Stevenson states that in the German Ocean, about 25 miles from Whalsey, mud is found upon the bottom in a depth of from 80 to 90 fathoms; in the latitude of Wick at from 60 to 70 fathoms; in the latitude of Kinnaird Head in 40 to 50 fathoms; in the Moray Firth at about 35 fathoms; at Dunbar in 22 fathoms. He states that the violence of wave action upon the shores of the German Ocean decreases in much the same proportion as the rise in level of the mud.

WAVES ON THE GREAT LAKES.

(*i*) The breakwater at Oswego Harbor, New York, consisted of a straight lake face 4,870 feet in length; a westerly shore return 916 feet in length, and an easterly shore return 246 feet in length. The breakwater was constructed of cribs 35 by 35 feet in plan and 20 feet in height, built of 12 by 12 inch timbers, and filled with stone, surmounted at low-water level by a continuous stone-filled superstructure, the banquette of which rose to about 5 feet above low water, and the parapet on the lake face, which was 12 feet in width, to about 13 feet above low water.

This breakwater was breached six times. Nearly all of these breaches originated in the substructure and appear to have been caused by the tendency of the lake face wall to pull away from the side walls. It was found that the greatest amount of pull was always from 5 to 8 feet below low-water datum, and that dislodgment invariably began at this level. If not repaired in time the dislodgment increased until the lake face wall was torn out and a breach occurred. These breaches did not extend lower than 12 feet below datum. The greatest depth of water in which the breached cribs were sunk was 25 feet.

(*j*) At Duluth, Minn., the old breakwater built out into Lake Superior, from near the foot of Fourth Avenue east in 1871–72, was partially destroyed by a severe northeast storm in November, 1872, and afterwards abandoned. This break-

water was composed of rock-filled timber cribs, and extended out into the lake to a depth of 26 feet. The top of the superstructure was 6 feet above the water. An examination in 1902 showed that as a result of 30 years of exposure to storms the lower portions of most of the cribs were still in place, but the upper portion of the entire breakwater had been carried away to a depth of from 2 to 10 feet below water.

(*k*) At the harbor of refuge, Milwaukee Bay, Wis., it was found as a result of one winter's experience that the limit of wave disturbance for half-ton stones was about 13 feet below the water surface, and for stones of 250 pounds more than 20 feet, the depth of the water being about 30 to 35 feet.

(*l*) After the completion of the rock-filled timber cribs forming the west breakwater at the upper entrance to the Portage Lake canals, Lake Superior, rectangular blocks of sandstone, 2.5 by 4 by 6 feet in size and weighing about 141 pounds per cubic foot, were placed alongside the cribs, as shown in Pl. VI, the depth of the water being about 20 feet, and the upper and lower surfaces of the stones being 16 and 18.5 feet, respectively, below low-water datum.

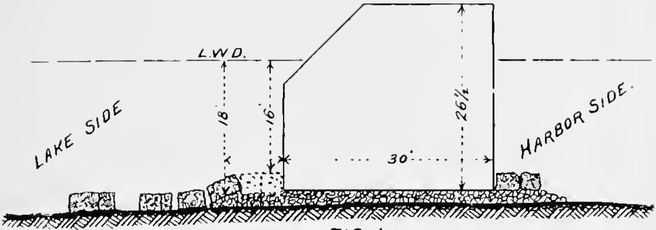
As a result of severe storms during the fall, winter, and spring of 1899–1900, nearly all of these blocks were displaced and carried lakeward—in some instances as much as 30 feet.

As the movement of these sandstone blocks was in a direction directly opposite to that of wave travel, it was undoubtedly caused either by reflected wave action or by the back draft of the mass of water thrown up by the wave.

(*m*) At Marquette, Mich., during a storm September 29, 1895, a piece of trap rock weighing about 600 pounds and lying on the bottom in water about 5 feet in depth, was lifted by a wave and cast upon the top of the unfinished portion of the vertical-faced breakwater, the bottom of the stone being at an elevation of 1 foot above the water level.

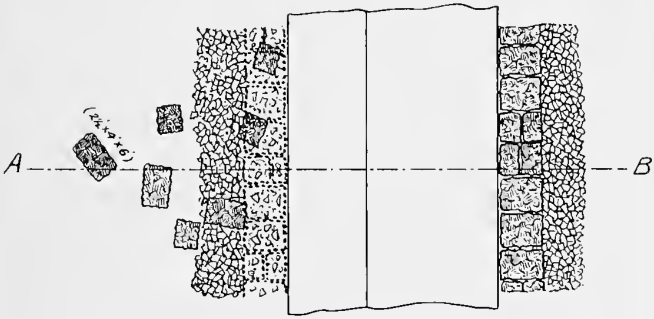
(*n*) At the same locality in January, 1896, several sacks of concrete weighing from 175 to 250 pounds, which had been lying on the bottom in water 8 to 10 feet in depth alongside the vertical face of a timber crib, were cast by the waves upon the concrete banquette of the breakwater, where they remained at an elevation of 5.5 feet above the surface of the water.

(*o*) During the storms just mentioned numerous irregular-



— FIG. 1. —

CROSS-SECTION ON A–B.



— FIG. 2. —

PLAN SHOWING BLOCKS OF SANDSTONE MOVED BY ACTION OF WATER DURING STORMS.

BREAKWATER AT UPPER ENTRANCE TO PORTAGE LAKE CANALS, LAKE SUPERIOR.

SCALE, 1 INCH = 25 FEET.

shaped pieces of trap rock, weighing from 75 to 100 pounds, and in water from 3 to 5 feet in depth, were thrown over the top of the concrete breakwater, the height of which was 10 feet above the surface of the water.

(*p*) At Grand Marais, Mich., on the south shore of Lake Superior, the west pier is composed of a foundation of fascine mattress work 50 feet in width, surmounted by a flat embankment of small stone, 3 feet in height. Upon the top of this embankment were sunk rock-filled timber cribs, rectangular in cross section, 24 feet wide and 16½ feet high; the deck of the cribs being 6 feet above low-water datum. The depth near the outer end of the pier was about 15 feet. Against the lake side of the cribs was placed an embankment of sandstone riprap, about 11 feet in height and 22 feet bottom width, composed of pieces of sandstone from 1 to 2½ tons in weight. This embankment reached to within 3½ feet of low-water datum.

The work terminated in 1897–98 and a careful examination made in 1902 showed that the entire riprap embankment on the lake side of the west pier, near its north end, had been washed away, and many of the pieces of riprap carried out as far as 30 or 40 feet from the side of the pier.

The heaviest storm waves travel parallel to the piers at this harbor, and the displacement in question was undoubtedly caused by breaking waves running alongside the west pier.

(*q*) The south pier at Superior Entry, Duluth-Superior Harbor, was strengthened in 1896 by an embankment of sandstone, the top of which was about 8 feet below low-water datum.

The storms of the two succeeding years flattened this embankment, which was covered with pieces of sandstone averaging about 1.6 tons in weight, so that its top varied from 10 to 14 feet below datum. In this, as in the previous case, the direction of wave travel is parallel to the piers.

(*r*) During the progress of dredging operations in Duluth-Superior Harbor a large quantity of dredged material, consisting of about equal parts of sand of medium fineness, and ordinary river mud, was deposited in Lake Superior, about 1.1 miles southeast of the outer end of the Duluth Canal in water of an average depth of about 45 feet.

The dumping of dredged material at this point terminated

on December 4, 1900, and an accurate survey of the locality was made through the ice in February, 1901. The result of this survey, given on Pl. VII, showed a least depth of 19.4 feet at the shallowest part of the dumping ground. Another survey made in a similar manner February and April, 1902, (see plate) shows remarkable changes during the year—the least depth having increased to 38.4 feet, and more than 100,000 cubic yards of material having been removed from the area shown on the plate and carried lakeward.

This result appears to have been due entirely to wave action, and to have been accomplished as follows:

During storms wave action stirs up the upper layers of sand and mud on the dumping ground, as can plainly be seen by the discolored water in the vicinity. While in this condition it is slowly and gradually carried lakeward by the undertow, which at the distance of five-eighths of a mile from shore, and in water 45 feet in depth, does not possess sufficient velocity to erode the dumped material unassisted by the wave action already described.

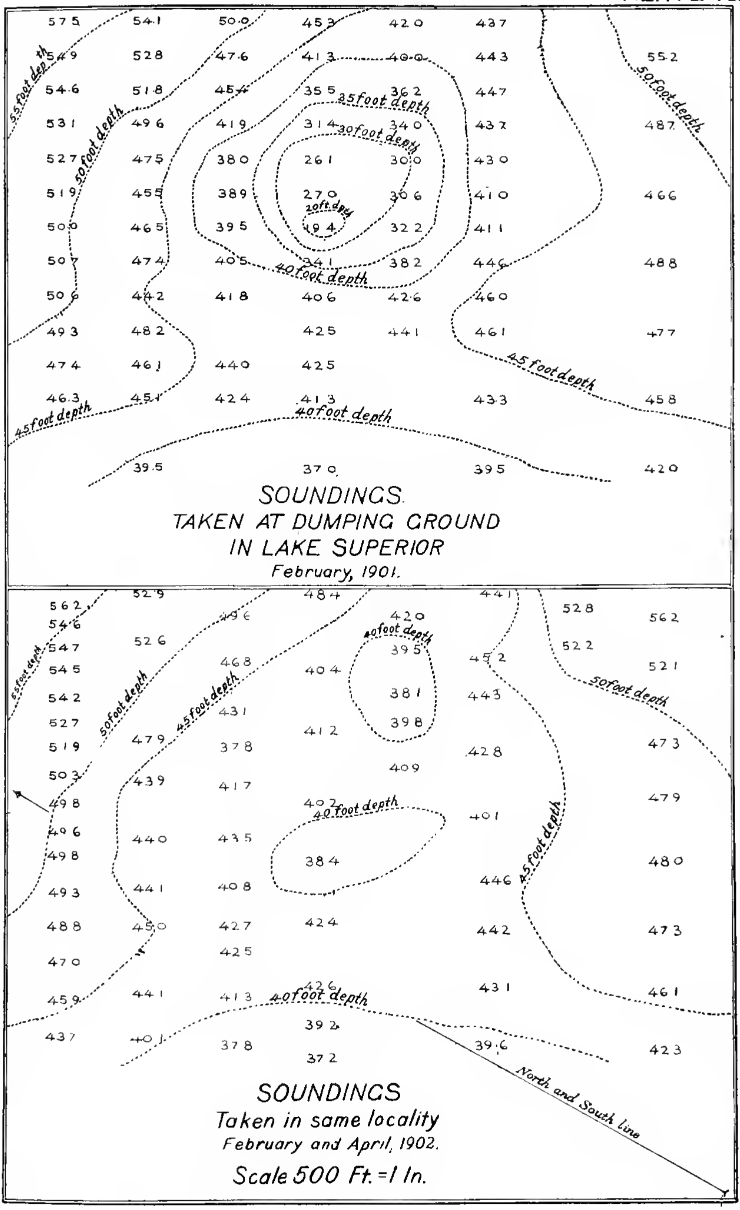
(*s*) During severe storms in the spring and fall sand is deposited by waves upon the deck of the piers at Grand Marais, Mich., and Duluth, Minn., the depth being 15 and 25 feet, respectively.

The same occurrence takes place on the breakwaters at the upper entrance of the Portage canals, and at Marquette, Mich., where the depths are 30 and 36 feet, respectively, showing that in all of these cases marked wave action extends to the bottom in the vicinity.

(*t*) In June, 1895, Mr. J. A. B. Tompkins, of Milwaukee, Wis., submitted to Mr. Charles Crosman, United States assistant engineer, a report as to the probable depth to which wave action extended, as derived from submarine observations made June 6 and 8 along the line of the intake pipe extension of the Milwaukee, Wis., waterworks, Mr. Tompkins having been permitted to accompany the inspection party sent out by Mr. George H. Benzenberg, city engineer of Milwaukee, to examine the condition of the intake pipe. The following are extracts from Mr. Tompkins's report:

The pipe commences at the crib, situated about three-fourths of a mile offshore, from the pumping station on North Point, and extends easterly a distance of about one mile; and beginning in a depth of about 25 feet

PLATE VII


55 foot depth
50 foot depth
45 foot depth
40 foot depth
35 foot depth
30 foot depth
20 ft. depth
SOUNDINGS
TAKEN AT DUMPING GROUND
IN LAKE SUPERIOR
February, 1901.
SOUNDINGS
Taken in same locality
February and April, 1902.
Scale 500 Ft. = 1 In.
North and South line

at the crib reaches a depth of 60 feet at its terminus. This locality is exposed to all, except westerly, storms, and is probably subjected to as violent a wave action as is to be found on the west shore of Lake Michigan. Previous to laying the pipe a trench was dredged to an average depth of 8 feet below the bottom of the lake, and of a width sufficient to admit of two lines of 60-inch pipe being laid therein, the dredged material being deposited to the south of the trench.

For a distance of about 1,500 feet from the crib the dredged material is sand, mixed with some coarse gravel; for the next 1,500 feet the bottom is very hard, with considerable shale; thence to the end of the pipe the material is clay, of varying degrees of hardness, and in places mixed with sand and gravel.

Up to a depth of about 44 feet there has been a very decided wave action, the ridge thrown up to the south of the trench having been washed down, and the trench generally half filled up. The dredging over this portion of the line was done in 1891–92. Beyond this depth there appears to have been but little action of the waves, scarcely any filling having taken place, except in a few places, where the trench has been filled from 1½ to 3 feet. This irregularity of filling, in marked contrast to what is to be found in the shallower water, may be due either to submarine currents rather than to wave action, or to the generally hard nature of the material on which the diminishing wave action would have but little effect, except at the softer spots.

That washing has taken place at depths as great as 55 feet (this being the depth of the lake bottom and not the top of the bank) there seems to be no doubt, as at this depth the pipe is about one-quarter buried, which would indicate that about 2 feet of filling had taken place, as the pipe was generally blocked up to overcome irregularities in the bottom of the trench. But that this wash has been caused by the action of the waves is open to doubt.

At the outer end of the pipe, in 60 feet of water, there are no indications of any disturbance of the dredged material. The banks, where the dredged material was deposited, maintain their irregular outline, and angular lumps of clay are to be found lying on the very edge of the bank, where the slightest movement of the water would displace them. This dredging was done in 1894.

A subsequent examination was made by the diver, under direction of Mr. Benzenberg, with special reference to determining the depth to which wave action extends.

Commencing at the outer end of the intake pipe, in 60 feet of water, the diver proceeded toward the shore, following along the ridge thrown up by the dredge. No indications of any disturbance of this ridge were to be seen until a depth of 50 feet was reached. At this point the diver reported that he thought he could discover indications of a washing of the bank. This depth of 50 feet refers to the top of the ridge, soundings being taken at the time of the examination. The ridge is about 4 feet high, or the bottom of the lake is 54 feet below the water surface. The material is sand and clay, with some stones, and is of a softer nature than the material found to the eastward or in the deeper water. Upon reaching this

point the examination ceased, it being assumed that this represented the maximum depth at which wave action had taken place.

While this last examination corroborates those previously made, to the extent, at least, that a washing and moving of material have taken place at a depth of at least 50 feet, it still fails to establish the fact that this washing was caused by wave movement produced during severe storms.

While the results of the examinations have not been wholly satisfactory and the matter is left rather indeterminate, there seems to be no doubt but that a decided wave action takes place at a depth of at least 40 feet. To what greater depths and to what, if any, extent wave action extends can only be determined by further observations, with special reference to this interesting subject.

(*u*) At the following harbors the change, from mud or clay bottom in deeper water to sand bottom on approaching shore, occurs at the depths given:

Duluth, Minn., Lake Superior, 55 to 60 feet; Chicago, Ill., Lake Michigan, 40 to 45 feet; Milwaukee, Wis., Lake Michigan, 40 to 45 feet; Cleveland, Ohio, Lake Erie, 33 to 38 feet.

In the preceding cases, unless geological or artificial conditions out of the ordinary impair the value of the indications afforded by the limiting depths at which mud or clay rests upon the bottom, these depths may be taken as defining the limits at which appreciable wave action upon the bottom ceases.

In the case of Milwaukee, Wis., it is interesting to note how this indication agrees with the depth, 44 feet, mentioned by Mr. Tompkins as defining the limit to which undoubted wave action extends.

CHAPTER X.

Measurements by means of spring dynamometers of the force exerted by waves. Observations by Thomas Stevenson. At Oswego Harbor, New York, 1884. At Milwaukee Bay, Wisconsin, 1894. At North Beach, St. Augustine, Fla., 1890–91. On Lake Superior, 1901–1903.

Dynamometer measurements of wave force by Thomas Stevenson.—The first and by far the most extensive series of dynamometer measurements of the force of waves are those commenced by Thomas Stevenson, the distinguished British engineer, in 1842, and carried on for many years thereafter. The instruments used by him were of the general type shown in the sketch, with disks from 3 to 9 inches in diameter, and

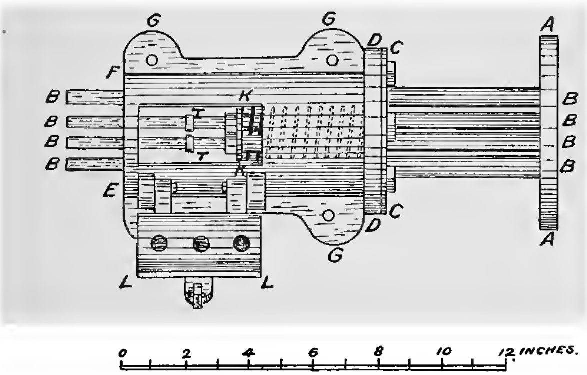


FIG. 11.—Thos. Stevenson's marine dynamometer.

with springs of strength varying from 10 to 50 pounds for every eighth of an inch of elongation. The observations were all reduced to the same value per square foot. The instrument was generally placed, unless otherwise specified, at about three-fourths tide, and the result obtained was the maximum reading for the period in question. Mr. Stevenson calls attention to the fact that the values obtained refer to areas of limited extent, and are applicable to the *piecemeal*

destruction of masonry, and should not be held to apply to large surfaces. In 1843 and 1844, in the Atlantic, at Skerryvore Rocks, and at the adjacent island of Tyree, the average reading for five of the summer months was 611 pounds per square foot, and for six of the winter months, 2,086 pounds. The maximum reading at Skerryvore was 6,083 pounds per square foot, the next highest being 5,323. At Bell Rock, in the German Ocean, the greatest result was 3,013 pounds; at Dunbar, 7,840 pounds; and at Buckie, the maximum reading from observations extending over several years was 6,720 pounds. Observations at Skerryvore Rocks and at the island of Tyree, in 1845, gave the following readings for waves supposed to be of the height given below:

Supposed height of waves.	Condition of sea.	Dynamometer readings.
Feet.		
6	Swell	3,041
10	Ground swell	3,041
20	Heavy sea	4,562
20	Strong gale, heavy sea	6,083

In 1858 observations were made by Mr. Stevenson at Dunbar Harbor to determine the relative effects of waves of translation and of oscillation against dynamometers fixed on an unfinished wall, and on isolated piles seaward of the wall. Two dynamometers were fastened to the isolated piles, and three were fixed on the unfinished wall. The depths at the time of high water in the vicinity of the dynamometers varied from 7 feet to 11 feet 5 inches. Waves from 1.33 feet to 3.75 feet reached the wall without becoming waves of translation, while waves of from 7 to 10 feet became breaking waves. The results showed that for waves of translation the mean force registered on the dynamometers attached to the wall was 2.01 times greater than on those attached to the pile. In the case of the oscillatory, or nonbreaking waves, the readings of the dynamometers on the wall was 22.45 times greater than those on the pile, showing that the broken surface of the unfinished vertical wall had served to develop the force of the waves. One of the dynamometers on the wall was placed in an angle formed by the junction of the old and new walls where its reading showed that the force was concen-

trated. If its indications be excluded from the result, the ratio for waves of translation for the dynamometers on the wall is 1.46 times greater than for those on the pile, and for oscillatory waves, 8.27 times greater.

In the preceding cases cited by Mr. Stevenson, the ratios given must be taken as mere approximations, for with the waves which he classes as "oscillatory," out of 17 possible readings from the dynamometers fastened to the piles, there were obtained 15 zero readings, one of 140 pounds per square foot, and one of 680 pounds, while the dynamometers fastened to the unfinished wall, out of 18 possible readings gave 15 readings varying from 264 to 4,053 pounds per square foot, and 3 of zero.

It is difficult to see how purely oscillatory waves of the dimensions given could register such pressures on the type of dynamometer used by Mr. Stevenson. This dynamometer is capable of recording only dynamic pressures, since static pressures against the exposed front and rear faces of the pressure plate would neutralize one another. The greatest dynamic pressure exerted by an unbroken oscillatory wave is that due to the orbital motion of the particles at and near the surface, which for waves of the size mentioned, would amount to but a few pounds per square foot at most, certainly not to as much as 140 or 680 pounds, which readings must have been due to the impact of waves which broke wholly or in part against the dynamometers fastened to the pile.

Dynamometer observations were taken at Dunbar in order to ascertain the relative forces exerted by waves at different levels. The dynamometers were let into the wall so that their disks were nearly flush with its face. (See sketch, p. 148.) Owing to an uncertainty regarding the readings of some of the instruments employed, which was not discovered until the results, which had extended over a period of years, were examined for reduction, Mr. Stevenson did not feel warranted in deducing from them any rule or formula. The observations, however, showed that by far the greatest force was exerted at the level of high water.

To determine the horizontal force of the back draft of the wave, dynamometers, with their disks *facing* the wall, were fastened to a pile immediately outside of the Dunbar bulwark, and others were fastened to the same pile with their disks

facing seaward. In a single instance described, the force of recoil was 2,240 pounds per square foot, while the direct force of the waves before reaching the wall was but 700 pounds, a result which Mr. Stevenson considered due to concentration of the filaments of water by the sea wall.

At the same time that the observations at Dunbar, just described, were made, two dynamometers were bolted to the top of the parapet, with their disks projecting over the edge and pointing downward, in order to ascertain the upward force of the ascending column of water and spray at the top of the wall. The maximum vertical force recorded by these dynamometers, which were 23 feet above the surface of the water

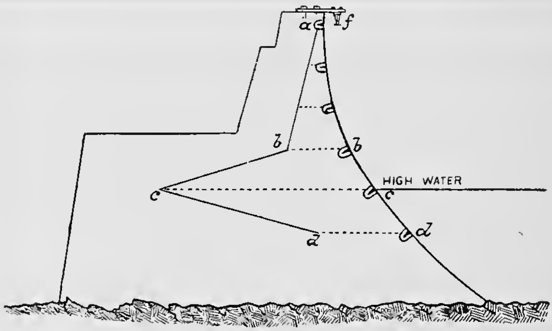


FIG. 12.—Sketch showing horizontal force at different heights, as determined by dynamometer measurements at Dunbar, by Thos. Stevenson.

at the time of observation, was 2,352 pounds per square foot, while the greatest horizontal force recorded by a flush dynamometer, fixed in the face of the wall 1.5 feet lower (see sketch above), was 28 pounds, showing that in this particular case the vertical force was about 84 times greater than the horizontal.

Experiments carried out with a dynamometer by Mr. Frank Latham on the sea wall at Penzance showed that with a depth of 10 feet of water the pressure of the water on the wall, due to the waves striking it at right angles, was from 1,800 to 2,000 pounds per square foot, the spray rising above the wall, which was nearly vertical, to a height of from 25 to 30 feet.

Dynamometer observations at Oswego Harbor, New York, 1884.—Dynamometer observations were made at Oswego Har-

bor, New York, in 1884, by Lieut. Col. Henry M. Robert, U. S. Corps of Engineers, and described by him as follows:

During the past season an effort was made to obtain data relating to the height, velocity, and force of the waves thrown upon the western part of the west breakwater.

Three dynamometers were attached to a vertical shaft and placed near the angle made by the shore arm with the main breakwater; one dynamometer was placed at the water surface, the second at 8 feet, and the third at 16 feet below the water surface. They were arranged so as to present their disks directly to the seas; the area of the disk was exactly one-half a square foot. The height of the waves was determined by a level placed upon the high lake bank west of the shore and directed lakeward upon the incoming waves. The height determined was that obtained by the wave when distant about 1,000 feet from the breakwater. The velocities were determined by the time required for the wave to sweep along the shore arm of the breakwater. The results of such observations may be summarized as follows:

During a severe gale from the northwest the waves attain a height of from 14 to 18 feet above the normal surface of the lake, with a velocity of from 30 to 40 miles per hour. The dynamometers placed at the surface of the water recorded a pressure of 400 to 600 pounds per square foot, while the dynamometers placed respectively 8 and 16 feet below the water surface gave no indications of even a pressure of 10 pounds per square foot. Late in the season a dynamometer was placed in proximity to the others, but attached to the pier, and at an elevation of 8 feet above water surface; but one reading was obtained in this position, which was 940 pounds per square foot. The gales from which these readings were obtained were far from the most severe, and it is highly probable that parts of the breakwater are subjected at times to a force of over 1,000 pounds per square foot. (Report of the Chief of Engineers, U. S. Army, 1885, p. 2279.)

The wave height was determined when the wave was traveling in water 24 to 30 feet in depth.

The total length of the shore arm of the breakwater was 910 feet, the depth of water at the outer end being about 18 feet, and decreasing fairly uniformly to zero at the shore.

The depth at the point where the dynamometers were located was about 18 to 20 feet.

Dynamometer observations at harbor of refuge, Milwaukee Bay, Wisconsin, 1894.—Dynamometer readings at the harbor of refuge, Milwaukee Bay, Wisconsin, were secured during the winter and spring of 1894 by Lieut. C. H. McKinstry, U. S. Corps of Engineers, whose description of the results obtained follows:

In the storm of February 12, 1894, the dynamometers were on top of crib No. 68 (a crib with superstructure), one on the harbor side and the other

with its disk flush with the lake face. They were, therefore, about $6\frac{1}{2}$ feet above water. The pressure registered by the first was more than 1,430 pounds per square foot, (how much greater the pressure actually was it is impossible to say; 1,430 pounds was all the instrument could record), and by the second less than 200 pounds. The wind was from the northeast; velocity, about 36 miles. In the storms of April 8–9, 1894, and May 18, 1894, both dynamometers were on top of crib No. 70 (a crib without superstructure), about $2\frac{1}{2}$ feet above water. In the first, the dynamometer on the harbor side read 3,460 pounds per square foot (all it could read), and the other read less than 300 pounds. Wind northeast to southeast; velocity, about 39 miles. In the second storm the dynamometers read respectively 1,970 and 316 pounds. Wind north to northeast; velocity, about 49 miles. (Report of the Chief of Engineers, U. S. Army, 1894, p. 2086.)

The cribs upon which the dynamometers were set were 100 feet long, 24 feet wide, $26\frac{1}{2}$ and $22\frac{1}{2}$ feet in height, respectively. They were sunk in water about 32 feet in depth. The bottoms of the cribs were about 12 feet above the original lake bottom and rested on a mound of stone 12 feet in height. In 1893 these and adjoining cribs were reinforced by depositing stone "so as to raise the riprap to within 13 feet of the water surface, with berms of 5 and 8 feet, and slopes of 1 on $1\frac{1}{2}$ and 1 on 1, on the lake and harbor sides, respectively."

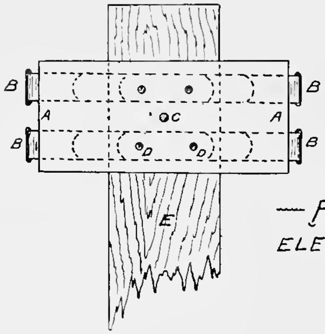
It is stated that the maximum height of waves is about 13 feet, but the report does not show whether this height is the result of measurements or is estimated.

In 1893 a dynamometer was set in the lake face of one of the cribs, the plate being 6 feet below the surface of the water, and no pressure was recorded at that depth.

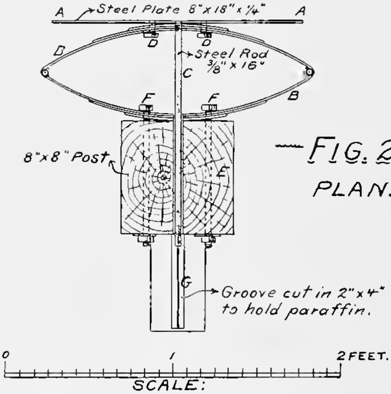
The dynamometers used were like the Stevenson dynamometer shown on page 145, except that a mixture of melted beeswax and paraffin, instead of the leather rings, was used to obtain the maximum readings, experience at St. Augustine, Fla., in 1890 having proved the former to be more accurate.

Dynamometer observations at North Beach, St. Augustine, Fla., 1890–91.—While engaged in constructing concrete groins to prevent the erosion of the shore on North Beach, St. Augustine, Fla., the writer undertook a series of observations upon "breakers," the only class of waves which could be observed in the vicinity of his work.

The results of the dynamometer observations only will be



—FIG. 1.—
ELEVATION.



—FIG. 2.—
PLAN.

DYNAMOMETER USED AT ST. AUGUSTINE, FLA. 1890-91.

given in the present chapter; those relating to wave height, velocity, etc., have been discussed in their proper places.

During the progress of the work of shore protection unusual facilities were afforded for observing not only the effect of the breakers on the work itself, but also on dynamometers placed in the immediate vicinity in various depths of water, and at various elevations above the water surface.

Experience with the sand-charged breakers of this coast made it almost certain that the marine dynamometer devised by Mr. Thomas Stevenson would, if set up here, become so choked by fine sand as to be inaccurate.

After some difficulty the type of dynamometer shown in the sketch was devised.

A steel plate AA, 8 inches by 18 inches (1 square foot) in area and one-fourth of an inch thick, was firmly bolted to two elliptical steel springs BB, similar to the best class of springs used under wagon seats. These springs were placed 6 inches apart from center to center. Their planes were parallel to each other and to the length of the plate, and at right angles to the plane of the plate. Protruding to the rear from the center of the plate, and perpendicular to it, was an iron rod C, three-eighths of an inch in diameter and 15 inches in length. The dynamometer when set up was fastened to the upright post E, 8 by 8 inches in cross section, by four 9-inch bolts F, F, two of which passed through each spring and then through the post. The iron rod passed through the post and about 1 inch beyond its rear face, terminating over a horizontal groove G. This groove extended nearly the entire length of a piece of 2-inch by 4-inch timber, mortised into the rear face of the post, and parallel to the iron rod. The groove was partly filled with a mixture of melted paraffin and beeswax, and a cover was lashed over it. The maximum compression during the interval between readings was recorded plainly and accurately on the surface of the paraffin by a small pointer on the end of the iron rod C. After the dynamometer was fastened to the post, but before it was set in place in the water, it was rated by piling weights on the plate and noting the corresponding compressions. Another rating made several months later showed no appreciable change in the elasticity of the springs. From time to time

the entire apparatus was covered with a coat of coal tar to prevent the corroding effects of sea water. Three dynamometers in all were used, but one of them was carried away in a heavy storm soon after it was set up. In no case was there the slightest difficulty in obtaining a clear and accurate measure of the maximum compression of the springs, even although, in a few cases, weeks elapsed between readings.

At the locality where the observations were made the shore line is straight and trends N. 13° W. The bottom is hard, smooth, and regular, and the depth from the shore seaward increases very gradually and uniformly, being 5 feet at 500 feet from shore; 10 feet at 1,700 feet; 15 feet at 3,300 feet; 20 feet at 5,500 feet, etc. As a result the direction of travel of waves is nearly always normal to the shore line, and, except in heavy weather, the breakers into which they are transformed on approaching the shore are quite regular, and their heights from hollow to crest, as well as their velocities, easily measured by means of graduated posts set near the groins. It is to these shore breakers exclusively that all observations taken in this locality apply. The period during which observations were made extended from January, 1890, to October, 1891. Owing to the rise and fall of the tide, all appreciable effects of direct wave action at the locality where observations were taken occurred between 2 feet above low water and 11.5 feet above the same plane. At no time during the progress of the observations was the depth of water greater than 6½ feet.

In all, 197 readings were obtained. Grouping the readings made when the conditions were most similar, the following table results:

TABLE XVII.—*Observations with spring dynamometers at North Beach, St. Augustine, Fla., 1890–91.*

Observed maximum wave dimensions.			Corresponding maximum dynamometer readings. (Pounds pressure per square foot.)	Number of dynamometer readings taken.
Height.	Length.	Velocity.		
Feet.	*Feet.*	*f. s.*		
2.0	46	8.4	148	3
2.5	60	9.4	230	12
2.75	70	10.3	269	5
3.0	75	11.7	322	20
3.5	78	12.0	313	8
4.0	82	12.2	406	20
4.5	90	14.0	452	15
5.0	120	15.2	467	16
5.5	130	16.7	550	11
6.0	150	18.2	667	7
Total				117

NOTE.—The dynamometers were set on 8-inch by 8-inch posts (braced at the rear), which were so located in the line of breakers that the instruments received the direct impact of the breaking waves.

The dynamometers used in these observations were of such strength that they were compressed about 1 inch for every 120 pounds of pressure.

Observations on Lake Superior, 1901–1903.—During the seasons of 1901–1903 the writer established spring dynamometers at several localities on Lake Superior, and secured 108 observations in all.

The dynamometers used are shown on Pl. IX. They consist of a pressure plate A, 12 by 12 inches in size; a flat spiral spring B of crucible steel; four recording rods C, riveted into the pressure plate A, and moving in the cavity G when the spring is compressed; the cast-iron bedplate D, which is fastened in the desired position by means of the bolts H, and the rubber buffers E, against which the round bolt heads F strike when the spring is released after compression. The recording rods C were covered with a mixture of melted wax and paraffin, which was scraped in passing through the bedplate D, and gave a record of the maximum compression since the rods had last been covered with wax and paraffin. A mean of the readings of the four rods was taken as the true reading. After the dynamometers had been some time in position they drooped a little at the outer end, and to remedy this two rods

parallel to the recording rods were permanently fastened into the bedplate D, so as just to touch the lower edge of the pressure plate A. These supporting rods were covered with melted wax and paraffin also, and gave the maximum distance that the lower edge of the pressure plate had moved inward, affording a check upon the readings of the two lower recording rods.

Each dynamometer was rated separately by static pressures before being set in place.

Dynamometers A, C, D, E, F, H, I, J, K, and L were of such strength that 1 inch of compression corresponded to a static pressure of about 885 pounds.

For M, N, and O, 1 inch of compression corresponded to a static pressure of about 1,065 pounds.

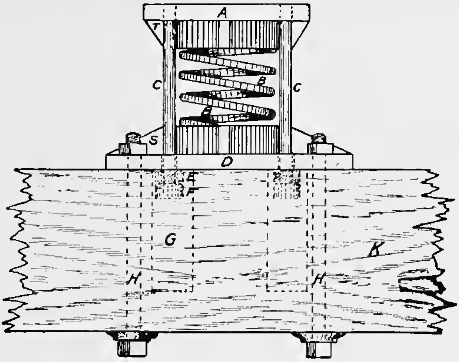
Except in the case of dynamometer M, they were each set up with the plate vertical and facing the direction from which the heaviest seas were expected.

The results of the observations are given in the following table, from which have been excluded only a few minor readings too small to be of value:

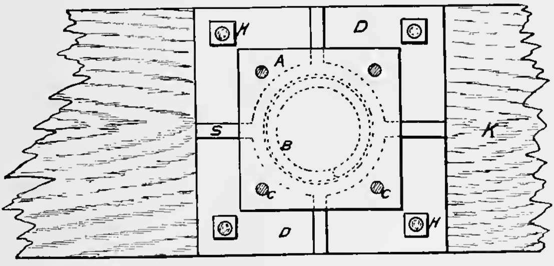
TABLE XVIII.—*Observations with spring dynamometers on Lake Superior, 1901–1903.*

SEC. I. OUTER END OF SOUTH PIERS, DULUTH CANAL AND SUPERIOR ENTRY.

Date.	Stage of lake above low water datum.	Observed maximum wave dimensions.			Corresponding maximum dynamometer readings, pounds pressure per square foot.			
		Height.	Length.	Velocity.	End of south pier, Duluth Canal—Elevation above low water datum at which dynamometers were set.			South pier, Superior entry.
					"C" +0.07′	"F" +3.74′	"A" +7.01′	"E" +3.74′
1901.	*Feet.*	*Feet.*	*Feet.*	*f. s.*				
July 24	+1.7	12	150	24.2	250	1,150	1,030	Not set.
Aug. 9	+1.9	12	130	24.2	370	1,075	780	1,190
Sept. 24	+1.9	16	250	33.2	1,630	1,930	2,050	2,255
								"D"
Oct. 9	+1.7	10	150	23.7	000	000	500	1,210
Nov. 6	+1.9	13	150	29.6	000	1,275	1,260	1,615
Nov. 22	+1.4	14	150	27.2	000	1,010	1,605	1,605
					"C"	"E"	"G"	
1902.					+7.04′	+12.57′	+16.18′	Removed.
Oct. 23	+1.5	13	200	30.0	800	445	000	Removed.
Oct. 25	+1.7	16	200	30.0	1,755	1,335	000	Removed.
Nov. 12	+1.7	18	250	32.0	2,370	2,195	1,370	Removed.
Dec. 20	+1.7	16	210	31.0	1,700	1,430	515	Removed.



— FIG. 1. —
SIDE ELEVATION.



— FIG. 2. —
FRONT ELEVATION.

COMPRESSION SPRING DYNAMOMETER USED ON LAKE SUPERIOR, 1901–2

— SCALE —

TABLE XVIII.—*Observations with spring dynamometers on Lake Superior, 1901–1903*—Continued.

SEC. II. ON EAST AND WEST BREAKWATERS, UPPER ENTRANCE, PORTAGE CANALS, MICHIGAN.

Date.	Direction of wind.	Stage of lake, above low water datum.	Observed maximum wave dimensions.			Corresponding maximum dynamometer readings (pounds pressure per square foot), dynamometers all set at +8.75′, low-water datum.		
			Height.	Length.	Velocity.	"K"	"L"	"J"
1901.		Feet.	Feet.	Feet.	f. s.			
Sept. 16	NW.	+1.2	....	....	....	2,525	Not set.	370
Oct. 2	NW.	+1.3	....	....	....	1,250	....do...	350
Oct. 12	NE.	+1.3	....	....	....	000	....do...	780
Oct. 15	NW.	+1.3	....	....	....	1,320	....do...	000
Oct. 17	NW and N.	+1.3	....	....	....	1,190	....do...	670
Oct. 31	NW.	+1.2	....	....	....	1,940	....do...	000
Nov. 5	NW.	+1.2	....	....	....	805	....do...	205
1902.								
Aug. 31	NW to N.	+1.1	....	....	....	1,135	610	800
Sept. 3	NNW.	+1.2	....	....	....	1,400	1,165	955
Sept. 11	NW.	+1.2	13.5	200	30.0	1,865	1,960	590
Oct. 7	NW.	+1.1	....	....	....	450	320	000
Oct. 13	N and NNW.	+1.0	....	....	....	410	315	270
Oct. 19	NW.	+1.1	....	....	....	1,345	840	000
Oct. 24	WNW.	+1.1	....	....	....	1,040	615	000
Oct. 26	WNW.	+1.0	....	....	....	2,235	1,560	700
Nov. 22	NW.	+1.0	....	....	....	1,830	1,605	760
1903.								
Sept. 13	NW.	+1.5	....	....	....	2,480	1,920	717
Sept. 23	NW.	+1.4	....	....	....	2,160	1,640	487
Oct. 3	NW.	+1.7	....	....	....	2,300	1,820	496

SEC. III. ON BREAKWATERS AT MARQUETTE AND PRESQUE ISLE AND ON BLACK ROCK, MARQUETTE BAY, MICHIGAN.

Date.	Stage of lake above low water datum.	Maximum dynamometer readings (pounds pressure per square foot).		
		"I" On breakwater at Marquette. El. = +4.3′.	"H" On breakwater, Presque Isle. El. = +10.75′.	"N" On Black Rock. El. = +3.0′.
1901.	Feet.			
October 9	+1.5	100	275	Not set.
1902.				
April 26	+0.4	000	340	Do.
October 13	+0.9	405	000	1,540
November 5	+1.1	000	000	1,275
November 13	+1.2	000	000	1,560
November 26	+1.2	000	000	1,160
December 3	+1.0	000	000	2,055

The elevation of each dynamometer as given in the preceding table refers to the elevation of the center of the pressure plate above low water datum.

The dynamometers on the south pier of the Duluth Ship Canal were placed vertically above one another on the sharp apex of the pier as shown in the photograph, page 156.

The three upper dynamometers were spring dynamometers, and the two lower ones, one of which is barely visible at the water surface, were diaphragm dynamometers, which will be described hereafter.

The depth, just in front of the dynamometers, referred to low water datum, was 22.5 feet, and the governing depth 200 feet lakeward, was about 25 feet.

The dynamometer at Superior Entry was fastened to a horizontal timber which projected from the end of the south pier, at right angles to the axis of the pier, and cleared the pierhead by several feet.

The result was that the dynamometers at the Duluth Canal and that at Superior Entry were struck by waves which, due to shoal water, broke just in front of them, and were unaffected by reflected wave action, or by any concentration of energy resulting from the form of the structure against which the dynamometers were fastened; in other words, the records were made under much the same general conditions as were those at North Beach, Florida.

The two breakwaters at the upper entrance of the Portage Canals, Michigan, make an angle of 90° with one another, the entrance channel being between their outer extremities. The direction of the east breakwater is northwest to southeast and that of the west breakwater is northeast to southwest, and the cross-section of the two is identical.

The locality is subject to as frequent storms as any other on Lake Superior.

The breakwaters are of the rock-filled timber-crib type. The horizontal deck of the superstructure is 19 feet in width and 8 feet above low water datum. The lake-face of the superstructure has a slope of 1 on 1, which slope extends 3 feet below datum. The rear wall is vertical. (See photograph, p. 223.)

Dynamometers K and L were set on the west breakwater facing northwest; the former at the junction of the sloping

SPRING AND DIAPHRAGM DYNAMOMETERS ON OUTER END OF SOUTH PIER, DULUTH CANAL.

face with the deck, and the latter on the deck, 15 feet to the rear and 10 feet to the side of dynamometer K. Dynamometer J was set on the east breakwater facing northeast, in a position corresponding to that of K. The depth in front of the dynamometers was about 29 feet.

In very severe storms the crests of the waves were higher than the deck of the breakwaters, and dynamometers K and J appeared to be struck by masses of water having some upward motion, while dynamometer L was struck by masses of water moving horizontally across the deck with such velocity as to be projected 25 to 30 feet in rear of the back wall before striking the water. On November 22, 1902, and September 13, 1903, measurements of this distance were made, and from these it was computed that the maximum horizontal velocity of the mass of water at dynamometer L was about 38 feet and 37.9 feet per second, respectively.

Dynamometers I and H were set on the breakwaters at Marquette and Presque Isle, respectively, the latter being set on the highest part of the structure, which is shown in the photograph on page 215.

No severe storm from the direction of principal exposure has occurred since these two dynamometers were set, and consequently their readings are of but little significance.

Dynamometer N was set 8 feet back of the water line and 3 feet above low-water datum, on a rock which is exposed to very severe wave action, and slopes down into deep water at a short distance from shore.

No unusually severe storm has occurred since this dynamometer was set in place, but from the readings so far obtained it is probable that its record will eventually exceed that of any of the other dynamometers.

It is interesting to note in the preceding table how nearly alike are the greatest dynamometer readings at each of the most exposed localities. For example, at Duluth Canal, Superior Entry, upper entrance of Portage Canals and Black Rock, the maximum readings are 2,370, 2,255, 2,525, and 2,055, respectively.

It is also interesting to note that in spite of popular belief to the contrary, the force exerted by waves against dynamometer K, near the front wall of the breakwater, was on an average about 30 per cent greater than that exerted against dynamometer L, near the back wall.

The fact that the back wall of a rock and timber breakwater is more apt to be damaged by wave action than the front wall is probably the cause of this belief, and is doubtless due to the fact that the back wall, unlike the front, has no mass of rock and timber behind it to aid in resisting the action of the waves. Especially is it weak as compared with the front wall, when the decking has been displaced by waves.

On noting the action of waves against a concrete sea wall which was built in the lake near the Duluth Ship Canal, in water of an average depth of about 3.5 feet, low-water datum, it was determined to attempt to ascertain the relative values of the horizontal and vertical forces exerted by the waves against the vertical face of the wall, the height of which was 10 feet above low-water datum.

To do this, dynamometer O was fastened to the wall with its center 4 feet above low-water datum, and the outer face of the pressure plate just 1 foot from the face of the wall. But a few feet away dynamometer M was set in place, face downward and horizontal, with the center of the pressure plate 10 inches from the face of the wall, and the lower face of the plate 3.7 feet above low-water datum.

The results of the observations secured with these dynamometers are given in the following table:

TABLE XIX.—*Observations with spring dynamometers fastened to sea wall, south of South Pier, Duluth Canal, Lake Superior.*

[Dynamometer O, face vertical. Dynamometer M, face horizontal.]

Date.	Stage of lake above low-water datum.	Depth at dynamometers on date given.	Corresponding depths 200 feet lakeward.	Maximum dynamometer readings (pounds pressure per square foot).	
				"O" Elevation =+4.00′.	"M" Elevation =+3.70′.
1902.	*Feet.*	*Feet.*	*Feet.*		
Aug. 18	+1.6	5.1	6.6	1,100	Not set.
Sept. 15	+1.3	4.8	6.3	000	610
20	+1.2	4.7	6.2	000	690
Oct. 13	+1.0	4.5	6.0	000	655
16	+1.2	4.7	6.2	000	770
23	+1.5	5.0	6.5	2,430	2,725
25	+1.7	5.2	6.7	000	1,100
Nov. 12	+1.7	5.2	6.2	1,160	790
Dec. 6	+1.3	4.8	6.3	1,060	Not read.
20	+1.7	5.2	6.7	2,490	1,100

It will be noticed that dynamometer M, measuring the vertical force, gave a reading for every storm, while O gave no reading whatever for 5 out of 10 storms. The largest single reading was due to vertical force, which was in excess of the horizontal in 6 out of 8 cases.

It is to be regretted that it was not practicable to set the pressure plate of dynamometer O flush with the face of the sea wall. If this could have been done the results might have been different, for on several occasions masses of water were seen projected rapidly upward along the face of the sea wall, the layer of water apparently being not more than a foot or so in thickness measured perpendicularly to the wall. This mass of water would impinge upon the pressure plate of dynamometer M, squarely and with considerable force, but would apparently produce no effect on dynamometer O. It is quite possible that the effect of the vertical wall, only a foot in rear of the pressure plate of dynamometer O, might have been to cause a counter pressure on the rear of this plate so soon after wave impact against the front, that during some storms no compression could take place. For this reason the types of spring dynamometers heretofore used do not appear well adapted for use against an area of large extent, and of a form which tends to prevent water in rear of the pressure plate from moving freely away.

It would seem from what has been stated, that they may, as a result of counter pressure, give readings too *small*, but can not give readings too *large*.

The effect of this sea wall was to produce concentrated wave energy at times, and this could plainly be seen in the case of the upward vertical force.

It is further shown by the fact that although the water just in front of the wall was never deeper than 5.2 feet, yet three dynamometer readings were obtained which were greater than the maximum reading at either the south pier of the Duluth Canal or at Superior Entry, and are on an average about four times greater than the maximum dynamometer reading for breaking waves of equal size at North Beach, Florida.

While the effect of this high vertical wall in shallow water was at times to concentrate wave action, yet waves were generally reflected by it to such an extent that during severe storms it received the impact of breaking waves much less

frequently than did the pointed ends of the concrete piers from which waves were never reflected.

The variation of the maximum dynamometer readings at the outer end of the south pier, Duluth Canal, according to the height of the instrument above the still-water level, is shown in fig. 13.

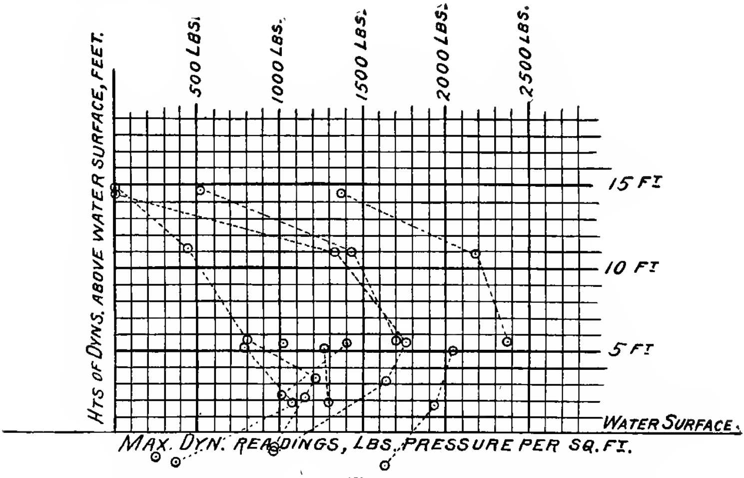


Fig. 13.

All readings connected by dotted lines occurred on the same date, but as spring dynamometers mark only maximum readings, it is not known whether the connected readings were made simultaneously. It is believed, however, as a result of the observations made with the diaphragm dynamometers that they were *not* made simultaneously.

CHAPTER XI.

Observations with Diaphragm Dynamometers.

Description of dynamometers and methods of rating same. Measurements of static and of dynamic pressures due to waves. Number of waves during a given period. Further dynamometer observations desirable.

DESCRIPTION OF DIAPHRAGM DYNAMOMETERS.

The spring dynamometers heretofore constructed have been quite limited in their scope, as they can not measure static pressures; they give only a single maximum reading for any one storm, and when placed against a vertical wall, or similar structure of considerable area, are possibly affected to some extent by back pressure. Moreover, were it not for the friction of the recording and guide rods, it is probable that the momentum of the moving parts of an instrument with weak springs, when struck by a wave, would carry it past its true position, indicating a pressure somewhat too great. With the strong springs used on Lake Superior in 1901–3, and the unavoidable friction of the supporting and recording rods, it is not believed that the readings secured were affected to any appreciable extent from this cause.

In order to secure an instrument which would register both static and dynamic wave pressures, and could be read for each individual wave, the writer in 1902 devised the type of dynamometer shown on Pl. X. In this instrument the flanged cylinder A is of iron and is cast in a single piece upon a bed plate, by means of which and the bolts KK, it is fastened in position. Over the flange of the cylinder is laid a diaphragm of rubber belting R, about a quarter of an inch in thickness. This diaphragm is held across the head of the cylinder by the flat cast-iron ring E, and the bolts FF, screwed tightly in place. Its exposed face is exactly one square foot in area. To prevent leakage, the faces of the flange and ring which are in contact with the diaphragm are machine planed.

At the highest point of the interior of the cylinder is a set screw S for letting out air when the cylinder is being filled. Into a projection cast on the back of the bed plate is screwed a wrought iron pipe PP, three-quarters of an inch in diameter, which leads, with two bends nearly at right angles to a small tank T located in an observing station on the south pier of the Duluth Canal. A half-inch branch pipe, with a downward inclination leads to the gauge G, which is of the modified Bourdon type, and registers pressures from 0 to 30 pounds per square inch. B and C are cut-offs in the pipes leading to the gauge and tank, respectively. All pipes and bends incline upward, so that no air will be retained in them after filling. When set for taking observations the gauge G was from 14 to 19 feet above the center of the diaphragm, and 6 to 12 feet from it horizontally. To lessen the momentum of moving parts, the interior of the gauge was filled with glycerin. When everything was in position, except the cut-off C and the tank T, the set screw S and cut-off B were opened, and the cylinder filled by pouring a small steady stream of water into the open end of the pipe below C until it ran out freely at S. The set screw was then screwed tightly in place and the filling continued until the pipe was filled to C. The cut-off C was then inserted and the pipe therefrom connected to the tank and the filling completed. C was then closed and B left open. Any pressure applied to the diaphragm was transmitted by the confined hydrostatic column to the gauge G, upon which it was read in pounds pressure per square inch.

For the first rating the instruments were filled and set up on land with the diaphragm horizontal, and with the same length of pipe and the same attached gauges as were to be used during wave observations. The pressure due to the head of water in the pipe produced a slight convexity in the diaphragm, and in order to secure uniform pressure on the latter when being rated, fine, dry sand was spread upon it and struck to a plane surface. A circular board just fitting the inside of the flat ring E was laid upon the sand, and on this weights were piled and the corresponding gauge readings noted.

The second rating was made by lowering the instrument foot by foot in still water, and noting the corresponding gauge readings.

The results of the two methods agreed very satisfactorily,

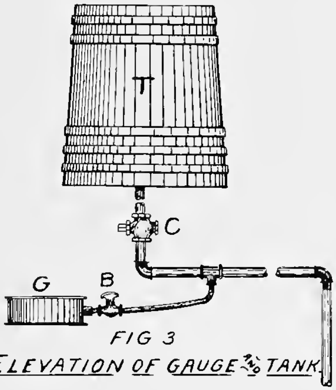


FIG 3
ELEVATION OF GAUGE AND TANK.

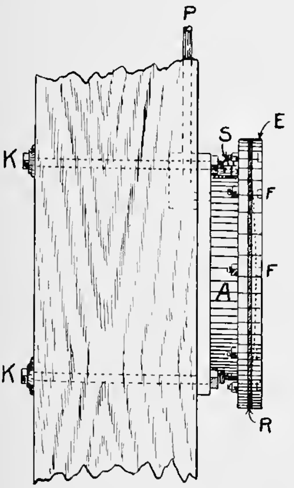


FIG. 2
SIDE ELEVATION.

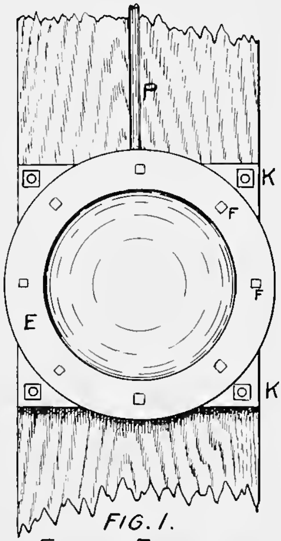


FIG. 1.
FRONT ELEVATION.

DIAPHRAGM DYNAMOMETER.

SCALE 3/4 INCH = 1 FOOT.

and when platted with the applied pressures as abscissas and the corresponding gauge readings as ordinates, gave what was practically a straight line.

In order to determine whether the instrument would give correct readings for pressures quickly applied and removed, it was placed on the ground with the diaphragm horizontal and a pressure was applied and removed, all in less than a second. The gauge was read and recorded at each application of the pressure. This was repeated ten times. For comparison, the same pressure was applied for half a minute at a time, giving a true static pressure. This was also repeated ten times. The aggregate of the ten gauge readings in the first case was eight-tenths of 1 per cent less than that in the second, showing that pressures applied and removed in a little less than a second could be correctly measured by this instrument. As the period of the ordinary storm wave is known to vary from about six to nine seconds, it was believed that it would be possible to obtain correct measurements of the pressures actually exerted by them with the diaphragm dynamometers just described. No difficulty was experienced from leakage, but to guard against this the cut-off C was opened and closed from time to time when the water was still or when a wave hollow left the diaphragm exposed in air. In all cases the index error was noted at the beginning and ending of observations.

MEASUREMENTS OF STATIC PRESSURES DUE TO WAVES.

For deep-water waves, as has been previously shown, Rankine has demonstrated mathematically that "the hydrostatic pressure at each individual particle during wave motion is the same as if the liquid were still."

It follows from this principle, and from the laws governing the orbital motion of particles, that the hydrostatic pressure upon a submerged body, not on the bottom, due to the crest of a passing wave, is less than that due to a column of water whose height is equal to the vertical distance from the submerged body to the highest point of the wave crest.

For the purpose of measuring the static pressures due to passing waves, diaphragm dynamometer No. 3 with its face horizontal and upward, was fastened on the berm of the inner wall of the south pier of the Duluth Ship Canal, with the center of the diaphragm 0.52 ft. below low-water datum. The berm in question was horizontal, 2 feet in width, and its

upper surface was 1 foot below low-water datum. It was formed by building a concrete superstructure 11 feet in height and 20 feet in bottom width upon a vertical-face timber crib 21 feet in height and 24 feet in width, the batter of the face of the concrete superstructure being about 1 inch in 1 foot.

Waves generally ran smoothly along and parallel to the piers, as may be seen from the photographs on pages 62, 63, 64, and 65.

The dynamometer could not be set in position until rather late in the season of 1902, and had to be removed on account of danger from freezing early in November, 1902, so that the observations secured were not nearly so full as desired.

The results of these observations are given in the following table:

TABLE XX.—*Observations of static pressures due to passing waves, Duluth Canal, Minnesota.*

Date.	Wave.		Height of wave crest above dynamometer.	Gauge readings (pressure per square inch).	d_0	Number of observations.	d'	Semi axes of orbits at depth. d'		Elevation of orbit centers at d'. η'	Head corresponding to static pressure on dynamometer.	
	Height.	Length.						a'	b'		From gauge reading.	Deduced theoretically.
1902.	*Feet.*	*Feet.*	*Feet.*	*Lbs.*	*Feet.*		*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*
Oct. 23	4.4	120	4.4	1.61	23.7	5	3.90	2.20	1.70	0.17	3.93	3.83
23	5.3	120	5.0	1.67	23.9	5	4.39	2.60	1.99	.29	4.07	4.29
23	7.2	125	6.4	1.85	24.3	4	5.35	3.45	2.55	.48	4.51	4.99
23	8.5	125	7.4	2.06	24.6	5	6.00	3.95	2.90	.62	5.02	5.43
23	9.6	140	8.4	2.20	25.1	1	6.79	4.71	3.19	.88	5.37	6.03
25	4.5	100	4.7	1.38	24.0	2	4.17	1.97	1.67	.14	3.37	4.13
25	5.5	110	5.5	1.54	24.2	23	4.70	2.48	2.00	.25	3.76	4.47
25	6.3	120	6.0	1.73	24.4	13	5.17	2.96	2.27	.37	4.22	4.96
25	7.3	135	6.8	2.10	24.6	5	5.69	3.67	2.59	.49	5.12	5.30
25	8.5	135	7.8	2.10	25.1	6	6.47	4.10	2.87	.72	5.12	5.91
25	10.5	150	9.2	3.23	25.5	1	7.35	5.22	3.35	.92	7.88	6.59
Total	......	......	421.9	......	......	70	......	......	......	......	300.79	337.84

NOTE.—Stage of lake above low-water datum, October 23, 1.44 feet; October 25, 1.75 feet. Elevation of diaphragm of dynamometer=0.52 foot below low-water datum. Depth at dynamometer=22 feet at low-water datum.

In the preceding table d_0, d', a', and b' are as explained for equations (13), (14), and (15), by means of which equations the values of a' and b' in columns 9 and 10 have been obtained, remembering that b_s is always equal to half of the observed wave height.

The elevation of orbit centers at d' is theoretically equal to $\frac{\pi a'b'}{L}$, but by observation it is found that the actual elevation of the centers of surface orbits is considerably greater than this, therefore the quantities in the eleventh column have been obtained by multiplying $\frac{\pi a'b'}{L}$ by the ratio of the observed to the theoretical elevation of the center of the surface orbit for each particular wave.

The hydrostatic pressure upon any particle which, when under the crest of the wave, is at the same elevation as the face of the dynamometer is theoretically the same as that upon the same particle when the liquid is still. The theoretical still-water position of such a particle is vertically below its position when at the highest point of its orbit by a distance equal to b' plus the elevation of the orbit center, and the theoretical pressure upon the face of the dynamometer resulting from the crest of the passing wave would be that due to a column of water whose height is the distance from the still-water surface to the position of rest of the particle which, when at the highest point of its orbit, is at the same elevation as the face of the dynamometer. The last column in the table gives the height of the static column just described.

Column 12 has been computed from column 5 by dividing the quantities in the latter by 0.41, which is the pressure in pounds per square inch for each foot of head, as determined for this particular instrument by rating.

It will be noticed from an examination of columns 4, 12, and 13 of the table that, considering the aggregate of the 70 observations, the actual measured pressures were but 89 per cent of the theoretical pressures and but 71 per cent of those which would have been due to the quantities in the fourth column had the latter been true static heads.

It is possible that the quantities in the fifth column were affected to some extent by the action of the 2-foot berm upon which the dynamometer was set, but it seems more probable that in shallow water the actual hydrostatic pressure from a wave is less than the theoretical, and this is indicated by the fact that the actual elevation of the orbit centers is almost invariably greater than the theoretical.

To trace the theoretical variation in hydrostatic pressure for any given wave as d' is increased, the following table has

been computed for a wave 200 feet in length and 14 feet in height, traveling in water of a depth of 26 feet, two-thirds of the wave height being above still-water level. From the data given $b_s = 7$ feet and $d_o = 28.33$. From Pl. I, or from equation (13) $a_s = 9.83$ feet.

The difference between any two quantities in the sixth column of the table is the measure of the hydrostatic pressure due to a layer of water under the wave crest, of a thickness equal to the difference between the two corresponding quantities in the seventh column.

All particles on the surface of a wave, whether in the hollow or on the crest, will be at the same elevation when the water is at rest. All particles on the bottom, supposed to be a horizontal plane, remain always at the same level. It therefore follows from the principle enunciated by Rankine that, theoretically, the hydrostatic pressure on the bottom is the same under the hollow as under the crest of a wave.

Table showing computed dimensions of orbits of particles, and still-water positions of particles under the crest, for a wave 200 feet in length and 14 feet in height in water 26 feet in depth.

d′	Elevation of orbit centers at d′.		Semi axes of orbits at depth d′.		Positions of particles whose orbit centers are at d′, referred to still-water surface (+above, −below surface).	
	Theoretical $\frac{\pi a'b'}{L}$	Actual (probable) elevations.	a′	b′	When water is at rest.	When particle is under crest of wave.
Feet.	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*
0.0	1.08	2.33	9.83	7.00	− 0.00	+ 9.33
2.0	.94	2.04	9.40	6.39	− 1.71	+ 6.72
4.0	.82	1.77	9.02	5.81	− 3.44	+ 4.14
6.0	.72	1.56	8.67	5.26	− 5.23	+ 1.59
8.0	.61	1.32	8.34	4.69	− 6.99	− 0.98
10.0	.53	1.14	8.08	4.20	− 8.81	− 3.47
12.0	.45	.97	7.83	3.70	−10.64	− 5.97
14.0	.38	.82	7.61	3.22	−12.49	− 8.45
16.0	.32	.69	7.41	2.72	−14.36	−10.95
18.0	.26	.56	7.26	2.26	−16.23	−13.41
20.0	.20	.43	7.14	1.83	−18.10	−15.84
22.0	.15	.32	7.03	1.38	−19.99	−18.29
24.0	.11	.24	6.96	.97	−21.91	−20.70
26.0	.05	.11	6.92	.47	−23.78	−23.20
28.0	.01	.02	6.91	.07	−25.69	−25.60
28.33	.00	.00	6.91	.00	−26.00	−26.00

MEASUREMENT OF DYNAMIC PRESSURES DUE TO WAVES.

To measure dynamic wave pressures, diaphragm dynamometers 1 and 2 were fastened to the outer end of the south pier of the Duluth Ship Canal, as shown in the photograph, page 156, the elevations of the centers of their diaphragms being 4.75 and 0.58 feet, respectively, above low-water datum.

Only two storms of any consequence were encountered while the dynamometers were in proper working condition, those of October 23 and 25, 1902, but a number of other observations were taken whenever fresh northeast winds caused waves which could be accurately observed. More than a thousand dynamometer readings for individual waves were obtained in all. The more important of these readings are given in the following table:

TABLE XXI.—*Observations with diaphragm dynamometers at outer end of south pier, Duluth Ship Canal.*

Date.	Stage of water, low-water datum.	Period occupied in observing.	Maximum corresponding wave dimensions.			Principal synchronous dynamometer readings (pressure per square foot).		Total number of readings recorded.
			Height.	Length.	Velocity.	Dyn. 2.	Dyn. 1.	
1902.	*Feet.*	*Minutes.*	*Feet.*	*Feet.*	*f. s.*	*Pounds.*	*Pounds.*	
June 17	+1.2	25	4.8	50	16.0	410	000	26
17						385	000	
July 2	+1.3	38	5.0	50	16.0	531	000	16
2						485	000	
3	+1.2	62	5.0	70	19.0	677	000	30
3						677	000	
Aug. 18	+1.6	91	6.0	90	21.3	864	....	79
18						864	000	
18						792	50	
18						720	....	
18						720	110	
18						648	000	
18						331	166	
18						315	166	
18						364	149	
Oct. 23	+1.5	81	10.0	150	25.0	965	....	292
23						821	....	
23						821	202	
23						821	000	
23						821	58	
23						677	101	
23						677	58	
23						677	22	
23						677	130	

TABLE XXI.—*Observations with diaphragm dynamometers at outer end of south pier, Duluth Ship Canal*—Continued.

Date.	Stage of water, low-water datum.	Period occupied in observing.	Maximum corresponding wave dimensions.			Principal synchronous dynamometer readings (pressure per square foot).		Total number of readings recorded.
			Height.	Length.	Velocity.	Dyn. 2.	Dyn. 1.	
1902.	*Feet.*	*Minutes.*	*Feet.*	*Feet.*	*f. s.*	*Pounds.*	*Pounds.*	
Oct. 23						461	1,210	
23						461	778	
25	+1.7	63	13.0	200	30.0	965	230	118
25						965	000	
25						821	144	
25						677	(a)	
25						677	302	
25						677	230	
25						677	000	
25						504	634	
25						389	634	
25						353	(a)	
25						317	(a)	
25						....	1,642	
25						....	634	
Total.	....	360	....	....	....	....	....	561

[a] The movement of the index hand of the gauge was so rapid for these three observations that the observer could not follow it with his eye quickly enough to secure accurate readings, but was of the impression that in each of these cases it corresponded to as much as 20 pounds per square inch, or, after correction for index error, 2,794 pounds per square foot. He was of the impression that the movement of the index hand in these cases was so rapid that it probably passed beyond its true position. For these reasons, and until further experiments can be made to determine this point, it has been considered best not to include them in the preceding table.

On June 17, July 2, and July 3, the waves were not high enough to reach the upper dynamometer, and consequently it gave no reading on those dates.

On August 18 the largest waves were just high enough to strike the upper dynamometer at long intervals, but in no case during observations was a reading on that dynamometer as great as the corresponding reading on the lower dynamometer.

On October 23 the maximum reading noted was given by the upper dynamometer, but only in two instances, during observations lasting for a period of eighty-one minutes, was the reading of the upper dynamometer greater than the simultaneous reading of the lower dynamometer.

During the period of observation on October 25, four read-

ings of the upper dynamometer were greater than the greatest reading of the lower dynamometer, and 10 readings out of a total of 46 noted on the upper dynamometer were greater than the corresponding readings on the lower dynamometer. Many of the smaller readings on the lower dynamometer and some on the upper dynamometer were undoubtedly due to static pressures.

On all dates the diaphragms were struck at times by waves which broke wholly or in part from the effects of winds or currents, and did not develop the same relative energy as do the larger waves which break in mass from shoal water.

On no date did the maximum observed reading of the upper dynamometer correspond to that of the lower, which would seem to indicate that for waves of the dimensions noted the maximum wave effects are somewhat limited in a vertical direction. This conclusion must not be accepted too hastily, for on none of the dates embraced in the table did the waves break as often and as compactly in front of the end of the south pier as they did on November 12, 1902. To avoid freezing, the pipes of both dynamometers had been filled with petroleum a few days previous to the date last mentioned, but during this storm the oil oozed out of the joints to such an extent that a full liquid column could not be maintained between the gauge and the diaphragm, rendering it impossible to secure accurate observations. The pressures transmitted by the combined oil and air columns were closely observed, however, and it was noticed that the readings of the two gauges were nearly the same for all of the larger waves, showing that on this occasion the pressure varied but little in a vertical distance of 4.17 feet. As single waves several hundred feet in length along the crest are of frequent occurrence, it is evident that the corresponding wave pressures may extend for a long distance in a horizontal direction.

TOTAL NUMBER OF WAVES DURING A GIVEN PERIOD.

Column 9 of the preceding table does not give all readings registered during the period stated in column 3, but only those of which a record was kept. Observations were made on three occasions to determine the total number of waves

registering on the lower dynamometer during a given period. The results of these observations were as follows:

Date.	Maximum wave dimensions.			Period occupied in observing.	Total number of waves registering on dynamometer No. 2.	Average interval between readings.	Wave period.
	Height.	Length.	Velocity.				
1902.	*Feet.*	*Feet.*	*f. s.*	*Minutes.*		*Seconds.*	*Seconds.*
July 3	5.0	70	19.0	10	120	5.0	3.6
August 18	6.0	90	21.3	8	56	8.6	4.2
October 23	10.0	150	25.0	53	232	13.7	6.0

It will be noticed that the number of waves reaching the dynamometer during a given period is considerably less than would be indicated by the wave period. This is invariably the case, and is doubtless due principally to wave interference. During all observations there are frequent periods of many seconds duration when no waves of appreciable size appear. See p. 49.)

As the study of the action of individual waves against dynamometers promises more valuable results to engineers than does that of single maximum readings for each storm, the writer hopes at some future time to continue observations with diaphragm dynamometers, using recording gauges for obtaining continuous pressures during an entire storm, and endeavoring to trace the vertical limits of wave effects by placing a number of diaphragm dynamometers one above another from a distance below the water surface to the necessary height above the same.

If opportunity offers, an attempt may also be made to discover how wave pressures are affected by (*a*) the area, (*b*) the inclination, and (*c*) the form of the surface against which impact takes place. This could be done in the first case by using plates or diaphragms of different areas, set with their centers at the same elevation, and facing in the same direction; in the second case, by giving plates or diaphragms of equal area and elevation different inclinations with respect to the face of a similar dynamometer set in the immediate vicinity, with its pressure plate vertical and facing the direction of wave travel. In the third case the pressure plates should be plane, cylindrical, hemispherical, wedge shaped, etc., the areas of projection against a vertical plane being the same in

each case and the centers being set at the same elevation. In all cases the instruments should be set as near together as practicable in order that conditions may be similar for all.

It is hoped that others will become interested in this important subject, which offers a wide field, as yet but scantily covered.

[NOTE.—Since the above was written, orders involving a change of station and the assumption of duties unconnected with works of harbor improvement will prevent further investigation of this subject by the writer.]

CHAPTER XII.

Nature of the wave force recorded by dynamometers. Measured pressures conform to those caused by a current against a plane. Injury to works usually caused by waves acting for an appreciable time. Wave impact does not resemble impact of a solid body. Experiments to determine law governing the pressure due to the projection of a mass of water against a disk. Maximum readings of spring dynamometers not caused by static pressures.

In order to determine the character of the force due to the direct impact of a wave against a dynamometer, it is necessary to consider what changes take place in wave motion at the instant of breaking. If a wave advancing into water of uniformly decreasing depth be carefully observed, it will be seen that the velocity and wave length are gradually decreasing, that the crest becomes narrower and steeper, and the hollow broader and shallower, while the height increases, for a time at least. The anterior slope of the wave becomes steeper and steeper, and finally when the depth which limits further unbroken propagation for that particular wave has been reached, at which time about 65 to 85 per cent of the total wave height is above still-water level, the top of the wave is thrown forward and falls upon the anterior slope, and the wave soon thereafter becomes wholly a wave of translation.

Observation will show that when the wave breaks, the mass of water which formed the top of the wave moves forward with a horizontal velocity at least as great, and usually greater, than that of wave propagation at the time of the breaking.

This can be shown by noting, as in the photograph on page 123, that the broken water from the crest is usually considerably in advance of the adjacent unbroken crest of the same wave. The writer has been unable to determine by observa-

tion the ratio of the horizontal velocity, v_c, of this mass of water to that of wave propagation, v, but from careful investigation of the subject it is believed that the maximum possible value of v_c, is equal to v plus the theoretical orbital velocity of a surface particle at the crest of the wave, which particle at this point is moving forward horizontally with the velocity v'' given in equation (21).

When the breaking wave is of considerable size, a large mass of water will impinge with nearly constant velocity for an appreciable time against the plate of a dynamometer opposed to it, and should register a pressure corresponding to the velocity v_c, with which the water is moving.

In such a case it would seem that the ordinary hydrodynamic formulæ for the pressure of a current on a plane surface normal to the direction of flow should apply, approximately at least.

Naval Constructor D. W. Taylor, U. S. Navy, in a work on the "Resistance of Ships," discusses the case of a submerged plane in a moving current of water as follows:

Before taking up in detail the components of a ship's total resistance I shall consider briefly the eddy resistance and skin resistance of a thin, flat, submerged plate. As it is convenient to discuss them separately I shall, in dealing with the eddy resistance, take the plate as frictionless and fully submerged.

In this condition let α denote the angle between the face of the plate and the direction of undisturbed flow of the water. Then we shall have in front of the plate nearly exact stream-line motion, while in the rear will be broken water and eddies. There will then be an excess of pressure on the front face causing a "head resistance," and a defect of pressure on the rear face causing a "tail resistance." Suppose we now fit behind the plate a frictionless solid such that, as the water comes around the edges of the plate, it flows off over the smooth solid with perfect stream motion.

The introduction of the solid would evidently make little or no change in the flow in front of the plate, or in the head resistance.

By the aid of the above artifice Lord Rayleigh has deduced the following formula for the total normal pressure on the front face of the plate:

$$P'_n = \frac{2\pi \sin \alpha}{4 + \pi \sin \alpha} \frac{w}{2g} . Av^2 .$$

In the above P'_n = the normal pressure in pounds, w = weight in pounds of a cubic foot of water, g = acceleration due to gravity in feet per second, A is the area of the plane in square feet, v is the speed of advance of the plane in feet per second, and α is the inclination of the face of the plane to the direction of advance.

The tail resistance can not be treated mathematically. It is commonly assumed that it follows the same law of variation as the head resistance—increasing as the square of the speed. This seems an allowable assumption for the majority of practical cases. It is evident, though, that if the speed be so great that the rear pressure is zero, then a further increase of speed will not be accompanied by less pressure or more tail resistance.

Since we can not treat the whole of the eddy resistance of a plate mathematically, it is necessary to have recourse to experiments, deducing from them more or less approximate formulæ to cover the ground. It should be noted that in making experiments eddy resistance will be mixed up with skin resistance, but the latter is comparatively small except at very small angles.

Reliable experiments upon the resistance of planes were made by M. Joessel in 1873. They were made in the river Loire, at Indret, near Nantes. Unfortunately the maximum speed of current in which experiments were made was only about 2½ knots.

M. Joessel aimed to determine—

1. The total normal pressure on a plane inclined at any angle.
2. The distance of the center of pressure, or line about which the plane would pivot, from the leading edge.

Joessel's conclusions as to normal pressure are expressed by the following formula, in which P_n denotes the total normal pressure (covering both head and tail resistance) in pounds, and the other symbols have the same meaning as in Rayleigh's formula:

$$P_n = 1.622 \frac{\text{Sin } \alpha}{(.39+.61 \sin \alpha)} \frac{w}{2g} . Av^2;$$

As to the center of pressure Joessel concluded that if l denote the breadth of the plane in the direction of motion, and x the distance of the center of pressure from the leading edge—

$$x = (.195+.305 \sin \alpha)\, l.$$

Joessel's total-pressure formula, while doubtless fairly accurate for moderately large angles of inclination, does not hold for small angles of 10° or less when the plane is advancing nearly parallel to itself.

Special and reliable experiments were made upon plates at small inclinations by Mr. William Froude. He gives the following as a fair expression for the normal pressure in fresh water upon planes about 3 feet wide:

$$P_n = 1.7 \sin \alpha\, Av^2.$$

Here the symbols have the same meaning as before.

Where the direction of undisturbed flow of the water is normal to the plate, the formula commonly used is—

$$P = f \frac{W}{2g} Av^2;$$

in which w, A, v, g are as before, and f is a constant to be determined experimentally, but with a limiting value which can not exceed 2.0.

For a fixed plane in a moving current of water Mariotte found f=1.25, and Dubuat found f=1.856.

Making the reasonable assumption that the maximum dynamometer reading on any date corresponds to the maximum observed wave dimensions at the same locality on the date in question, we can test the applicability of the last formula to those dynamometer observations contained in Tables XVII, XVIII, and XXI, for which the maximum wave dimensions were observed.

Substituting $v+v''$ for v in the formula, and taking as the weight of a cubic foot of salt water 64.4 pounds, and of fresh water 62.4 pounds, and remembering that the area of the pressure plate used was unity, the values of f and f_m given in the following table result:

TABLE XXII.—*Comparison of observed and computed maximum readings of spring dynamometers.*

SEC. I. NORTH BEACH, ST. AUGUSTINE, FLA., 1890–91.

Maximum wave dimensions.				$(V+V'')^2$	$f=\frac{2gP}{w(v+v'')^2}$	f_m	Maximum dynamometer readings (pressure per square foot).		Number of observations.
Height.	Length.	Velocity.	V'' (computed).				Observed.	Computed.	
Feet.	*Feet.*	*f. s.*	*f. s.*				*Pounds*	*Pounds*	
2.0	46	8.4	2.9	127.7	1.16	......	148	168	1
2.5	60	9.4	3.2	158.8	1.45	1.69	230	209	12
2.75	70	10.3	3.5	190.4	1.41	1.69	269	249	5
3.0	75	11.7	3.7	237.2	1.36	1.80	322	310	20
3.5	78	12.0	4.0	256.0	1.22	1.48	313	335	8
4.0	82	12.2	4.1	265.7	1.53	2.00	406	348	20
4.5	90	14.0	4.9	357.2	1.27	1.81	452	469	15
5.0	120	15.2	5.3	420.2	1.11	1.80	467	550	16
5.5	130	16.7	5.7	501.8	1.10	1.64	550	657	3
6.0	150	18.2	6.2	595.4	1.12	1.44	667	780	7
Total	....	....	....	....	....	......	....	....	107
Mean	....	....	....	....	1.32	1.71	....	....	......

TABLE XXII.—*Comparison of observed and computed maximum readings of spring dynamometers*—Continued.

SEC. II. DULUTH CANAL AND SUPERIOR ENTRY, LAKE SUPERIOR, 1901–02.

Maximum wave dimensions.				$(V+V'')^2$	$f=\frac{2gP}{w(v+v'')^2}$	f_m	Maximum dynamometer readings (pressure per square foot).		Number of observations.
Height.	Length.	Velocity.	V'' (computed).				Observed.	Computed.	
Feet.	*Feet.*	*f. s.*	*f. s.*				*Pounds.*	*Pounds.*	
12.0	150	24.2	7.3	992.2	1.20	......	1,150	1,259	1
12.0	130	24.2	7.9	1,030.4	1.20	......	1,190	1,307	1
16.0	250	33.2	10.6	1,918.4	1.22	......	2,255	2,432	1
10.0	150	23.7	6.0	882.1	1.41	......	1,210	1,118	1
13.0	150	29.6	9.8	1,552.4	1.07	......	1,615	1,968	1
14.0	150	27.2	9.6	1,354.2	1.23	......	1,605	1,718	1
16.0	200	30.0	10.3	1,624.1	1.12	......	1,755	2,059	1
18.0	250	32.0	11.5	1,892.2	1.29	......	2,370	2,400	1
16.0	210	31.0	10.4	1,714.0	1.02	......	1,700	2,174	1
Total	....	....	....	....	....	......	....	....	9
Mean	....	....	....	....	1.20	......	....	....	....

SEC. III. UPPER ENTRANCE, PORTAGE CANALS, LAKE SUPERIOR, 1902.

13.5	200	30.0	8.5	1,482.2	1.36	......	1,960	1,879	1
....	....	....	....	1,444.0	1.31	......	1,830	1,831	1
....	....	....	....	1,436.4	1.38	......	1,920	1,824	1
Total	....	....	....	....	....	......	....	....	3
Mean	....	....	....	....	1.35	......	....	....	....

Mean value of f for all observations = 1.31.

The maximum dynamometer readings given in the eighth column are the means of the entire number of maximum readings indicated in the last column. The values of f given in the sixth column correspond to these means. The values of f_m given in column 7 are deduced from the highest single maximum reading in each case.

The quantities in the ninth column have been computed from the formula

$$P=\frac{fwA(v+v'')^2}{2g}, \tag{24}$$

giving to f the value 1.31 and to the other quantities the values already stated.

It will be noted that the mean value of f deduced from the preceding observations is nearly the same as that determined experimentally in running water by Mariotte, but is considerably smaller than the values determined by Rayleigh, Joessel, Froude, and Dubuat, which are probably more nearly correct.

It will be seen upon examining column 7, Section I, of the

table, that in one case only out of 107 observations did f_m reach its maximum possible theoretical limit of 2.0, yet in three other cases it equalled or exceeded 1.8. It should, therefore, always be borne in mind when using equation (24) that f_m *may* have a possible value as great as 2.0.

For comparison with the preceding table, the maximum observations with diaphragm dynamometers 1 and 2 have been arranged in the same manner as were those with spring dynamometers. The results are shown in the following table:

TABLE XXIII.—*Comparison of observed and computed maximum readings of diaphragm dynamometers on outer end of south pier, Duluth Canal, 1902.*

Maximum wave dimensions.				$(V+V'')^2$	$f=\frac{2gP}{w(v+v'')^2}$	Maximum dynamometer readings (pressure per square foot).	
Height.	Length.	Velocity.	V″ (computed).			Observed.	Computed.
Feet.	*Feet.*	*f. s.*	*f. s.*			*Pounds.*	*Pounds.*
4.8	50	16.0	4.7	428.5	0.99	410	542
5.0	50	16.0	4.9	436.8	1.26	531	553
5.0	70	19.0	4.2	538.2	1.30	677	681
6.0	90	21.3	4.6	670.8	1.33	864	850
10.0	150	25.0	6.3	979.7	1.28	1,210	1,241
13.0	200	30.0	8.5	1482.2	1.14	1,642	1,877
Mean..	....	....	....	....	1.22	....	....

It will be noticed that the mean value of f is almost identical with that previously deduced for the spring dynamometers at North Beach, Florida, and on Lake Superior, apparently showing that equation (24) applies in this case also.

The last column of the table has been computed from this equation, giving to f, as in the previous case, the value 1.31.

It will be seen from what precedes that observations with spring dynamometers taken at North Beach, Florida, where the maximum compression of the spring was 5.5 inches, and on Lake Superior, where it was 2.8 inches, the spring being in one case about seven times stronger than in the other, follow the same law as those taken at the latter place with diaphragm dynamometers, the exposed faces of which had no movement in the direction of the applied wave force.

This clearly indicates, as will be shown hereafter, that the *measured* wave impact, except possibly in the case of the three doubtful observations described on page 168, in no wise resembled the impact of a solid body, but apparently followed the law of dynamic pressure due to flowing water.

It is quite possible that wave impacts will be encountered which are so brief in their action that it will not be practicable to measure accurately the resulting pressures with any form of dynamometer yet employed.

Such impacts are believed to be ordinarily of little importance to the engineer, and seldom to represent the maximum forces which assail a work.

Careful observation indicates that these sudden "slaps" against a structure are almost invariably caused by comparatively small masses of water, in some instances combined with imprisoned air, which, owing to reflection or to wave interference, strike the work with abnormal velocity, throwing spray into the air and making a loud report, but seldom causing serious damage.

This phase of wave action is always most marked when a storm is increasing in violence, or when the wind has just attained its greatest velocity, the surface of the water in such cases being in a state of great confusion, rendering it difficult to distinguish any regular system of waves. Such a condition is shown in the photograph on page 216. This chaos of waters which so impresses the beholder simply represents *dissipation of energy*, and usually causes but little damage to completed works.

On a large body of water, when the wind begins to subside, the crests of the waves are no longer blown over, the larger waves overtake the smaller ones and combine with them, and a regular system of waves of larger size predominates in place of the many confused systems previously existing.

These large waves of regular form, like that shown opposite page 64, represent *concentrated energy*. Where such a wave breaks in front of a work a large mass of water rushes forward (sometimes on Lake Superior for as long as three or four seconds) with great velocity and, sweeping against a structure for an appreciable time, may decrease its weight by submergence, overcome its inertia, and displace or damage the parts against which it impinges.

It is the almost invariable experience of engineers engaged upon marine works that serious damage to such works seldom occurs when the wind velocity is greatest, but is usually inflicted after the wind has subsided to a marked extent.

While the writer's own experience confirms entirely the opinion just stated, it should be remembered that local peculi-

arities of a particular site may cause the work to encounter its greatest exposure to wave action at some other period of the storm than that mentioned. For instance, if the governing depth for some distance in front of a work is about 20 feet, the work would probably be exposed to attack by most waves under 12 feet in height, by few greater than 16 feet in height, and by none greater than 20 feet in height, and if the particular locality permitted the formation of waves of height as great as 20 feet, the work might suffer least when the waves outside were highest.

As a result of what precedes, it may be stated that the greatest damage to a marine work is usually caused by a large mass of water rushing against it for an appreciable time, causing a pressure which appears to conform to that indicated by well-established hydrodynamic laws for a current flowing against a submerged plane.

It will now be further shown—

(1) That the impact of a wave does not resemble that of a solid body.

(2) That a mass of water in air projected with a certain velocity against a plane surface can produce no greater pressure than would be caused by the steady flow against this surface, of a jet of equal cross-section having the same velocity and striking at the same angle.

(8) That the pressures indicated by spring dynamometers, as usually constructed, are due to dynamic action only.

Experiments by Thomas Stevenson to show that the impact of a wave does not resemble that of a solid body.—The accuracy of his dynamometer results having been questioned, Mr. Stevenson in 1844 set up at Skerryvore three dynamometers having disks of different areas and springs of different strengths, for the purpose of obtaining simultaneous observations under similar conditions.

The means of 9 sets of observations made with two instruments were 433 pounds and 415 pounds, respectively. The means of 18 sets of observations made with all three instruments were 1,247 pounds, 1,183 pounds, and 1,000 pounds, respectively; a degree of correspondence which, considering the character of the force measured, must be regarded as satisfactory. It having been claimed that the action of a wave is much the same as the impact of a hard body, Mr. Stevenson showed, from the results just quoted, that if they had so acted the same force would, in different marine dyna-

mometers, assume different statical values, according to the space in which the force is expended, so that with the same force of impact the indications of a weaker spring would be less than that of a stronger. In order to throw light upon this subject, a cannon ball weighing 32.5 pounds was dropped from the same height upon the disks of two dynamometers. The strength of spring in one dynamometer was 462.24 pounds per inch of elongation, and in the other 156 pounds. The results of these experiments are shown in the following table:

Experiments by Thomas Stevenson showing effect of impact of a cannon ball upon dynamometers having springs of different strengths.

Height fallen by cannon ball.	Calculated velocity at impact.	Elongation of spring, dynamometers—		Registered pressure of dynamometers—		Ratio of recorded pressures—dynamometers A and B.
		A.	B.	A.	B.	
Feet.	*f. s.*	*Inches.*	*Inches.*	*Pounds.*	*Pounds.*	
0.5	5.67	0.875	1.5	404.5	234	0.58
1.0	8.02	1.25	2.0	577.8	312	.54

Strength of spring per inch of elongation, dynamometer A=462.24 pounds; dynamometer B=156 pounds.

It will be seen from these experiments that although the force of impact was precisely the same in each case for the two springs, yet the pressure indicated by spring B, the weaker spring, was but little more than half that shown by spring A.

It seems clear, therefore, that if the waves had acted by impact similar to that of a solid body, Mr. Stevenson would not have obtained the harmonious results at Skerryvore with the three dynamometers in the simultaneous measurements previously described. From these experiments he was convinced that "the action of a wave, therefore, is not momentary, as in the case of a solid, but is continuous during the period that the disk is immersed in the passing wave," and that there is no valid objection against statical determination of the dynamic effect of a wave, when in marine structures this dynamic action is usually opposed by the dead weight of masonry. He was, therefore, convinced that "the dynamometer furnishes exactly the kind of information which the engineer requires."

Experiments at Duluth, Minn., 1902.—In further investigating the question of the reliability of dynamometer measurements of wave force, it was considered desirable to deter-

mine whether the sudden impact or blow of a body of water could cause a greater pressure than that due to a continuous flow at a velocity equal to that of impact. To do this the experiments hereafter described were planned and carried out with the efficient assistance of Mr. J. H. Darling, United States assistant engineer.

Apparatus.—A section of fire hose with a thirteen-sixteenths-inch nozzle was coupled to a city hydrant, where the ordinary pressure was about 105 pounds per square inch, and could be regulated at the nozzle by an intermediate valve. The nozzle rested on a timber frame which was adjustable, so that the jet could be thrown horizontally and exactly upon the pressure disk *d*.

The resulting pressure was measured by a contrivance consisting of a spiral spring *s*, of which two sizes were used—one for greater and one for smaller pressures. Through the spring passed a wooden spindle, keyed so as to compress the spring when pressure was applied to the disk at the end.

This disk was circular in form, with a flat surface, and measured exactly one-half of a square inch in area, which was less than the area of a cross section of the jet. The latter, therefore, always fully covered the disk. The wooden spindle was three-eighths inch in diameter, 15½ inches in length, and passed through two bearings. The rear end carried an index which moved along a scale marked to indicate pressures in pounds. This index was a slender pencil lead, made to trace a record of maximum and minimum pressure on a card with an elastic backing, so as to yield as the pencil was adjusted to bear against it and receive a light and uniform mark. The index and the observer were protected from the flying water by suitable screens. For all steady pressures the pencil was drawn back so as not to make a tracing, and the pressure was read off by eye. The instrument was rated by turning it to a vertical position and loading the disk with weights. With the weaker spring the compression was found to be nearly proportional to the weight. For the stronger spring the compression was not quite uniform, and the scale was graduated accordingly.

The weight of spindle, spring, and disk was 2 ounces, and with the spindle vertical in the operation of rating this weight acted with the loading weights, but did not act with the water pressures during the experiments, as the spindle

was horizontal. Due allowance for this was made in reducing the observations.

The velocity of the water in the jet was determined as follows:

Immediately before or after experiments at any fixed rate of flow the jet was discharged into a barrel for a definite period, usually from twenty to thirty seconds, and the volume of discharge computed by weighing the barrel before and after the jet was turned into it. With the known area of cross section of the jet and the volume and period of discharge, the velocity was computed with a limiting error not exceeding 2 per cent. The limiting error in the case of the measurement of pressure was somewhat greater, varying with each spring from about 2 per cent for the greater velocities to about 8 per cent for the smaller.

Pressures from continuous streams.—The pressures from continuous and steady streams, from the nozzle against the disk at various velocities, are given in Table A.

Column 2 of the table gives the theoretical pressure per square inch computed from the formula

$$p=f'\frac{w}{144}\cdot\frac{v^2}{2g}; \qquad (25),$$

(f' being assumed as unity), which for fresh water becomes $p=0.0067v^2$, the weight of a cubic foot of water, $=w$ being taken as 62.4 pounds.

TABLE A.—*Pressure of jet from a nozzle directed horizontally and striking a flat disk normally.*

[Orifice circular, 0.81 inch diameter; area, 0.52 square inch. Disk circular, 0.80 inch diameter; area 0.50 square inch. Length of jet, 6 to 12 inches.]

Velocity of jet.	Theoretical pressure per square inch.	Observed pressure per square inch.	Ratio of observed pressure to the theoretical.	
Feet per sec.	*Pounds.*	*Pounds.*		
15.5	1.6	1.4	0.88	Light spring.
24.3	4.0	3.6	.90	Do
27.0	4.9	4.4	.90	Do
31.0	6.4	5.6	.88	Do
33.2	7.4	6.8	.92	Do
33.8	7.7	6.8	.88	Do
42.3	12.0	12.2	1.02	Heavy spring.
73.0	35.7	34.2	.96	Do
82.8	45.9	40.2	.88	Do

Pressures due to sudden impact from an interrupted jet.—Having determined, as previously described, the pressures from steady and continuous streams, experiments were made to compare such pressures with those due to sudden impact of the jet against the disk.

For giving impact a shutter, *A*, was used to intercept the stream which was running continuously from the nozzle, the latter fixed in position and directed truly against the disk *d*. The first form of shutter used was a small rectangular board, with flushings on three edges, to prevent splashing of water, mounted at the end of a lever which was pivoted near its middle. The farther end of the lever was moved by hand, so as to shut off the stream and let it on again quickly. This was found to be too slow for the best results. A preferable arrangement was a simple board one-fourth inch thick, 5 inches wide, and 4 feet long, which was passed rapidly across the stream along a vertical guide, *G*, placed a few inches in front of the disk. The board was given a rapid downward motion from a considerable height, like the blow of an ax. This shut off the stream from the disk for the brief interval while the board was passing, and let it on again as the shutter cleared the jet. The attempt to cut the stream off squarely, and thus give an instantaneous and simultaneous blow of the entire cross-sectional area of the stream against the disk, could not be fully realized on account of insufficient speed of shutter. The actual velocity given the shutter probably did not ordinarily much exceed the jet velocity used, and would therefore give the jet an oblique front.

In the first experiments the index which was left free dropped to zero while the water was shut off, and upon impact sprung back to a point slightly beyond that due to a continuous stream of the same velocity. This result was due to the momentum of the moving parts of the apparatus, and to avoid this the spring was held at the point of steady pressure by an adjustable stop in front, but was free in each case to move to the rear, and indicate a greater pressure. Several repetitions of the cut-off were made at each velocity, but in not a single instance did the sudden impact of the interrupted jet give a reading in excess of that due to steady and continuous flow at the same velocity.

Table B gives the results of these experiments.

TABLE B.—*Conditions of tests of sudden impact of interrupted jet; spring and index held at the reading for steady flow at the same velocity.*

Velocity.	Length of jet.	Pressure on disks for steady stream; impact no greater.	Distance of face of shutter from the constrained position of disk.	Shutter.	Remarks.
Ft. per sec.	*Inches.*	*Pounds.*	*Inches.*		
15.5	9	0.7	3.1	C.	Light spring C is board $\frac{1}{4}$ by 5 inches by 4 feet.
27.0	9	2.2	4.5	C.	Light spring.
	9	2.2	8.2	C.	Do.
	36	1.9	4.2	C.	Do.
	36	1.9	25.9	C.	Do.
33.8	9	3.4	5.6	C.	Do.
	9	3.4	6.4	D.	D is 1 by 6 inches by 3 feet; light spring.
42.3	6	6.1	3.0	C.	Stiff spring.

With the lowest velocity used—15½ feet per second—it was possible readily to inspect the form of jet. After impinging against the pressure disk, it was found to spread into a conical sheet of water of well defined and beautiful form, the water being perfectly limpid and the surfaces smooth and regular. The surface began to turn outward about five-sixteenths of an inch before reaching the disk. The amount of deflection of the outer elements of the conical surface from their original direction was 57° 27′.

The experiments given in Table B, which embrace velocities ranging from 15.5 to 42.3 feet per second, apparently indicate conclusively that the sudden impact of a mass of water against a plane surface in air does not produce a greater pressure than that due to continuous and steady flow at a velocity equal to the velocity of impact.

Pressure due to volume of water falling upon a spring dynamometer.—In order to reproduce as closely as practicable the conditions which exist when a mass of water falls vertically upon a horizontal surface, as the deck of a breakwater, and to measure the force which it exerts upon this surface, a series of observations was made in March, 1903, with the apparatus shown in fig. 14, the disk *a* being horizontal and the lighter spring being used. The disk was circular and exactly half a

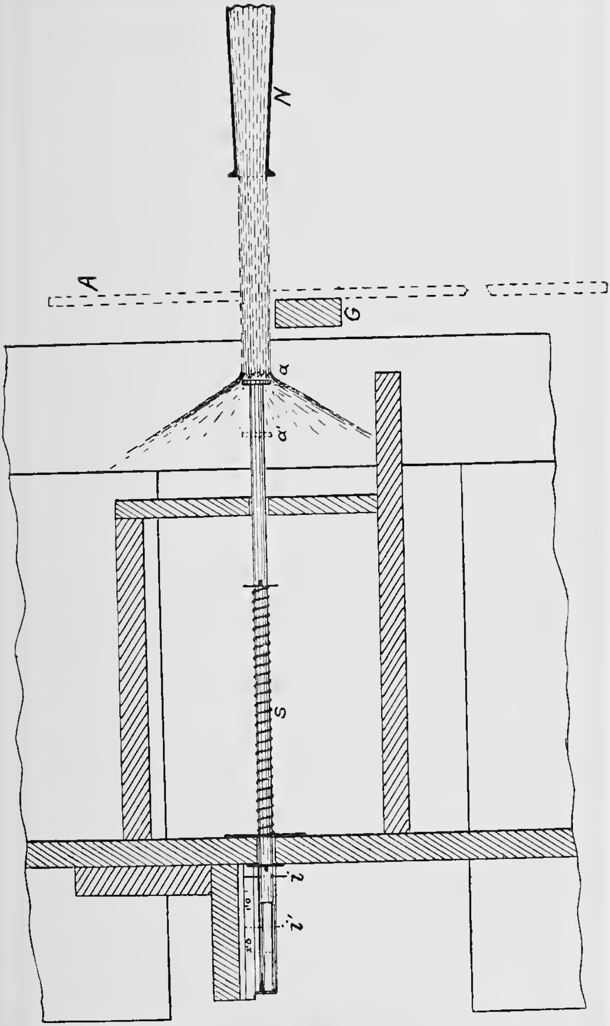


Fig. 14.

square inch in area. The spring used was compressed nearly an inch for each pound of static load. A volume of water varying from 2.5 to 3.5 gallons was dropped from heights of 5 feet and 9 feet, respectively, upon the disk *a* by quickly upturning the containing vessel.

The vertical thickness of the mass of water at the instant when it impinged upon the disk varied, as nearly as could be judged, from about 2 inches to 2 or 3 feet.

The volume of water being small it was usually considerably "broken up" before striking the disk, especially in the case of the greater drop, and consequently the pressures recorded were usually much less than the theoretical static pressure of a column of water the height of which is the same as that from which the mass of water was dropped upon the disk. Twenty-five observations were made for the 5-foot drop, and 125 for the 9-foot drop.

The results are given in the following table, which contains only the largest readings in each set of observations. Many of the readings were quite small, as it was not always practicable with the method employed to strike the disk squarely.

Pressures due to a volume of water falling upon the disk of a spring dynamometer from known heights.

Height of drop.	Quantity of water used.	Number of observations.	Static pressure due to a height equal to the drop.	Greatest observed impact readings.	Ratio of observed to theoretical pressure.
Feet.	*Gallons.*		*Pounds.*	*Pounds.*	*Per cent.*
5	3.5	7	1.083	0.95	88
				.85	78
				.80	74
5	3.5	7	1.083	0.95	88
				.95	88
				.90	83
5	3.5	11	1.083	0.70	65
				.60	55
				.56	52
9	2.5	8	1.95	0.95	49
				.80	41
9	2.5	12	1.95	1.15	59
				1.05	54
9	2.5	13	1.95	1.00	51
				.95	49
9	3.5	13	1.95	1.60	82
				1.50	77
9	3.5	9	1.95	1.80	92
				1.50	77
9	3.5	10	1.95	1.40	72
				1.20	63
9	3.5	10	1.95	1.45	74
				.92	47
9	3.5	11	1.95	0.60	31
				.55	28
9	3.5	9	1.95	0.54	28
				.50	26
9	3.5	10	1.95	1.15	59
				1.15	59
9	3.5	10	1.95	1.50	77
				1.40	72
9	3.5	10	1.95	1.35	69
				1.25	64
....	....	150	....	....	....

It will be noticed that, at heights both of 5 and of 9 feet, single pressures were obtained which were but little less than the pressures corresponding to static heads equal to the distance through which the volume of water fell, while in not a single case out of 150 observations was this theoretical pressure reached.

Pressure due to volume of water falling upon a diaphragm dynamometer.—For the purpose of comparison, it was decided to continue these experiments, using a modification of the diaphragm dynamometer instead of the instrument used in the previous experiment.

As it was impracticable to drop a volume of water large enough to cover the entire diaphragm (1 square foot in area) of the instruments used in wave observations, the cylinder A and its diaphragm R were replaced by a short length of $1\frac{1}{16}$-inch pipe joined to the pipe p by an elbow, so that at its outer extremity it was vertical. Over the end of this pipe was fastened a horizontal diaphragm consisting of two sheets of rubber of a total thickness of 0.04 of an inch. The recording gauge was filled with glycerin, and the whole apparatus was set up in the same manner as when wave observations were being taken. A mass of water was dropped upon the diaphragm, as in the previous case, and the gauge readings noted. Both before and after these readings were taken, the apparatus as a whole was rated by submerging the diaphragm to known depths in still water, and noting the corresponding readings. With these are compared the readings obtained by dropping a volume of water from a height equal to the depth of submergence.

As nearly as could be judged, the vertical height of the volume of water striking the diaphragm varied in these experiments from a few inches to 4 or 5 feet.

TABLE D.—*Pressures due to a volume of water falling upon the disk of a diaphragm dynamometer from known heights.*

Height of drop.	Quantity of water used.	Number of observations.	Gauge reading for static pressure due to height equal to drop.	Greatest observed gauge readings for impact.	Ratio of observed to theoretical pressure.	Remarks.
Feet.	*Gallons.*		*Divisions.*	*Divisions.*	*Per cent.*	
5	3.25	10	1.5	1.0	67	Poured quickly from round bucket.
				.5	33	
				.5	33	
5	2.75	12	1.5	1.5	100	Poured rapidly from coal scuttle with round lip.
				1.2	80	
				1.1	73	
5	2.75	8	1.5	.8	53	Do.
				.7	47	
				.7	47	
5	2.75	8	1.5	1.5	100	Dropped or poured very rapidly from same.
				1.5	100	
				1.0	67	
5	2.75	9	1.5	1.5	100	Do.
				1.5	100	
				1.4	93	
8	2.75	12	2.4	1.8	75	Poured rapidly from coal scuttle with round lip.
				1.8	75	
				1.7	71	
8	2.75	18	2.4	2.8	117	Dropped or poured very rapidly from same.
				2.4	100	
				2.2	92	
8	2.75	14	2.4	2.3	96	Do.
				2.2	92	
				2.0	83	
....	....	91	....	....	....	

It will be noticed by comparing the last two tables that for a drop of 5 feet the mean of all results given in the sixth column is 75 per cent for the spring dynamometer, and 73 per cent for the diaphragm dynamometer, and that the observed reading equaled the theoretical in 5 out of 47 observations with the diaphragm dynamometer, but in not a single instance with the spring dynamometer, the maximum observed value for this instrument being 88 per cent of the theoretical.

For a drop of 9 feet the mean of all results given in the sixth column is 58 per cent for the spring dynamometer, and for an 8-foot drop, 89 per cent for the diaphragm dynamometer. The observed reading of the diaphragm dynamometer equaled the theoretical in one instance, and exceeded it in

another, out of 44 observations in all, while in the case of the spring dynamometer the maximum observed value was 92 per cent of the theoretical.

From what precedes it will be noticed that considering all experiments described, the observed reading exceeded the theoretical in but a single instance out of 216 observations taken. In this particular case the movement of the gauge index was unusually rapid, so rapid in fact, that it could scarcely be followed by the eye, and it is quite possible that the momentum of the moving parts of the gauge carried the index beyond its true position.

In no case could the duration of the pressure upon either disk or diaphragm have exceeded a third of a second.

The experiments just described apparently confirm what has previously been shown, namely that the impact of an unconfined mass of water will not give a greater pressure than that due theoretically to steady flow at the velocity of impact.

As the velocities used in the experiments described cover the velocities of ordinary storm waves both upon the ocean and on the Great Lakes, and as the mass of water thrown forward by a breaking wave has ordinarily a velocity not greatly in excess of that of the wave itself, it would seem from these experiments that the impact of such a mass could properly be measured by a suitable device for recording pressures.

To show the effect of the impact of a solid body upon the springs used in the experiments previously described, symmetrically shaped pieces of iron, varying in weight from 1 to 16 ounces, were dropped upon these springs from a height of 1 foot, the corresponding compressions being recorded automatically. The results are given in the following table:

Compression of springs of different strength caused by dropping weights upon them from a height of 1 foot.

Weight dropped.	Compression.		Corresponds to static pressure.	
	Spring A.	Spring B.	Spring A.	Spring B.
Pounds.	*Feet.*	*Feet.*	*Pounds.*	*Pounds.*
0.0625	0.07833	0.0283	1.36	4.12
.1250	.1117	.0400	1.82	5.81
.2500	.1667	.0575	2.60	8.31
.5000	.2417	.0825	3.66	12.00
1.0000	....	.1200	....	17.25

Strength of spring A=1.4 pounds per inch of compression.

Strength of spring B=12.1 pounds per inch of compression.

An inspection of the preceding table shows that although the force of impact was identical for the two springs, yet the compressions of the stronger spring corresponded to static pressures more than three times greater than those of the weaker spring. Both in the case of continuous flow and of sudden impact of the jet of water, as shown in Tables A and B, the resulting pressures were practically unaffected by the particular strength of spring employed in the experiments.

It would therefore appear that the impact of a mass of water exerts a true pressure, and does not produce at all the same effect as that of a solid body.

Indicated dynamometer pressures not due to static head.—It has sometimes been contended that the pressure exerted at any height by a wave can not exceed that due to the head corresponding to the height of the wave crest; or, in other words, that the pressure due to the wave is entirely a static pressure.

An inspection of the type of dynamometer used by Thomas Stevenson in Great Britain, and of those used by the writer at St. Augustine, Fla., and on Lake Superior (see fig. 11 and Pls. VIII and IX), will show that in each case the rear of the plate is almost as fully exposed to static pressure, when the crest of the wave is passing, as is the front; and consequently, if this pressure alone existed, it would act practically to the same extent, but in opposite directions, both upon the front and rear of the thin vertical pressure plate. The two pressures would thus balance one another, and the spring dynamometers described could give no reading differing much from zero. As these dynamometers have repeatedly given pressures as great as several thousand pounds per square foot, it is evident that these records can not result from static pressures.

At North Beach, Florida, in 1890, the dynamometers were observed during several entire tides, and a record kept, among other things, of the maximum dynamometer reading and the height above the center of the dynamometer plate of the crest of the highest wave, or breaker, striking it during the tidal period.

The results of these obervations are shown in the following table:

TABLE XXIV.—*Dynamometer readings at North Beach, Florida, 1890.*

Date.	Reference of— Water surface.	Reference of— Center of dynamometer plate.	Reference of— Crest of breaker.	Height of breaker.	Crest of breaker above center of dynamometer plate.	Dynamometer readings.	Static pressure corresponding to a water column of height of crest above center of plate.
1890.	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Pounds.*	*Pounds.*
January	4.0	4.8	7.0	4.0	2.2	460	141
Do	4.0	4.8	7.0	4.0	2.2	472	141
Do	3.5	4.8	6.2	3.5	1.4	340	90
April	3.0	5.0	5.2	3.0	.2	197	13
Do	3.5	5.0	6.2	3.5	1.2	340	77
May	3.7	5.0	6.4	3.7	1.4	366	90
Do	4.0	5.0	7.0	4.0	2.0	433	128
Do	4.1	5.0	7.1	4.1	2.1	447	134
Do	3.0	3.0	5.2	3.0	2.2	355	141
Do	2.7	3.3	4.8	2.7	1.5	228	96
Do	3.1	3.4	5.4	3.1	2.0	380	128
Do	3.6	3.2	6.3	3.6	3.1	516	198
Do	3.5	3.2	6.2	3.5	3.0	472	192
June	2.0	2.2	3.8	2.5	1.6	210	102
July	5.1	7.5	9.0	5.1	1.5	501	96
August	4.5	7.5	7.9	4.5	.4	440	26
Mean				3.6	1.75	385	112

From the preceding table it will be noticed that the dynamometer readings correspond to pressures nearly three-and-a-half times greater than any possible theoretical static pressures, due to water columns of the height of the crest of the highest breakers above the center of the dynamometer plate; but, as already explained, a dynamometer of the type used here can not record any static pressure; neither is the pressure under the crest of a wave as great as that of a water column of corresponding height.

CONCLUSIONS.

From what precedes it may be concluded—

(1) That the impact of a wave does not at all resemble that of a solid body.

(2) That the pressures indicated by the types of dynamometers heretofore used are due to dynamic action only.

(3) That these pressures apparently conform to the hydrodynamic laws governing the action of a current flowing normally against a submerged plane.

(4) That a mass of water in air projected with a certain velocity against a plane surface can produce no greater pressure than would be caused by the steady flow against this surface of a jet of equal cross section having the same velocity and striking at the same angle.

From what has just been stated, it apparently follows by inference—

(5) That a mass of water projected against a submerged plane surface of considerably smaller area than the cross section of the mass can produce no greater pressure than would be caused by the steady normal flow at the same velocity of a current against a submerged plane surface of equal area and similar in form to the first.

(6) That as the most destructive waves act for an *appreciable period*, the pressures which they exert can properly be measured by suitably constructed dynamometers.

CHAPTER XIII.

Comparison of theoretical and observed wave force at Wick, Scotland; North Beach, Florida; Milwaukee, Wis.; Buffalo, N. Y.; Marquette, Mich.; Upper Entrance Portage Canals, Michigan; Duluth, Minn. Movement by waves of material on the bottom, at Duluth, Minn.; Upper Entrance Portage Canals, Mich.; Marquette, Michigan; Milwaukee, Wis.

In the present chapter it is proposed to compare observed and computed wave effects in all available cases where sufficient data are given.

It must be understood that the solutions which follow are approximate only, and were made with the idea that by means of them it might be possible to determine whether in these cases the observed effects could have been produced by the measured or computed wave forces.

In considering the different instances cited, it must be remembered that the deduced approximate value of the *minimum force* required to produce any observed effect of wave action is usually not the limiting value of the force which actually did produce the effect noted, but is probably considerably smaller. For example, a force which moves masonry blocks of a certain size may be capable of moving similar blocks of two or three times the size were they exposed to its action. In cases such as those to be described we would therefore expect that the force *available* was probably considerably in excess of that actually exerted in producing the particular effect noted. As the reading of a spring dynamometer is supposed to mark the maximum effect which the wave force is capable of exerting at that particular point during an entire storm, it follows from what precedes that these readings should ordinarily be in excess of values deduced from observed effects.

EFFECTS DUE TO OCEAN WAVES.

Damage to Wick breakwater, 1872 (for detailed account see p. 127 and Pl. IV).—The weight of the mass of concrete moved bodily was about 1,350 long tons. The exposed face presented

to the sea was about 546 square feet. The breakwater had on various occasions been assailed and damaged by waves estimated by the resident engineer to be 42 feet in height, which passed over the top of the parapet, which was 21 feet above high water, in solid masses of water estimated to be from 25 to 30 feet in depth. If we assume, as is probable, that the bed joint at CD was not water-tight, that the submerged concrete weighed 72 pounds per cubic foot, and that the frictional resistance of the mass to sliding was 75 per cent of its submerged weight, we can compute the minimum mean pressure per square foot on the exposed face which would just move the mass.

Making the necessary computations, this pressure is found to be 2,015 pounds per square foot.

It will be remembered that the concrete mass just considered rested on two rows of concrete blocks, the blocks in the upper row being 5 by 12.5 feet in cross section and those in the lower row 6 by 10.4 feet. All blocks in the upper row were carried away by waves after the superincumbent mass of concrete had been moved, while none in the bottom row were disturbed.

Computing, as in the previous case, the minimum pressure which would just move a single block in each row, we find that for the upper row it is 675 pounds per square foot and for the lower 562 pounds.

If we assume that the individual blocks in each row received no support from friction against adjacent blocks, which may have been approximately true for those on the inner edge of the work, the figures just deduced show that the pressure per square foot against the blocks in the upper row exceeded 675 pounds, while against those in the bottom row it was less than 562 pounds. The centers of blocks in the upper row were 23.5 feet below the level of high water of spring tides and those in the lower row 29 feet below the same plane. The mid height of the exposed face of the superincumbent mass of concrete was 4.5 feet below this plane, and the pressure per square foot against the lowest part of its exposed face must have been considerably greater than the mean pressure, 675 pounds, against the face of the blocks in the row next beneath it.

The length of the largest storm waves at Wick is not stated, but it is known from what has been shown previously that an ocean wave 42 feet in height is usually not less than 500 feet in length. Assuming this length, the corresponding theoretical wave velocity for a depth of 46 feet is 38.7 feet per second and the orbital velocity of a surface particle 17.6 feet per second. Substituting these values in equation (24), and making f equal to 2.0, we have for the maximum possible pressure per square foot of the wave 6,340 pounds.

Displacement of concrete blocks and riprap, North Beach, Florida, 1890–91.—At North Beach, Florida, in 1890, a block of concrete 2.5 by 6 by 10 feet, weighing 21,600 pounds, and resting on a flat bed of rubble 1 foot thick, was lifted vertically at least 3 inches, when it was caught and held fast. A dynamometer, located but a few feet away, at an elevation of 0.5 foot above high-water level, registered a maximum pressure of 633 pounds per square foot on this occasion. The area of the base of the block was 60 square feet, and the weight of the concrete of which it was composed was 144 pounds per cubic foot. The depth of the water was 6 feet, and the block was entirely submerged, the plane of the base being but 1 foot above the bottom. The elevation of this block was caused by pressure freely transmitted through the numerous voids in the rubble base upon which it rested. The minimum theoretical mean pressure on the base which could move the block is shown by computation to be 200 pounds per square foot.

A concrete block 2.5 by 2.5 by 5 feet, weighing 4,500 pounds, was moved 12 feet horizontally and overturned on one of its longer edges. The center of the block was 4.8 feet above the bottom, and at high-water level before being moved. A spring dynamometer in the immediate vicinity, set with the center of the plate 1.4 feet above high-water level, indicated on this occasion a maximum pressure of 533 pounds per square foot.

The minimum computed mean pressure per square foot which would have overturned the block if entirely submerged is 200 pounds, and which would have moved it laterally, assuming a coefficient of friction of 0.75, is 150 pounds.

A block of concrete 2.2 by 2.2 by 5.2 feet, weighing 3,600 pounds, and with its center at high-water level, was moved

laterally several inches by waves which did not exceed 4 feet in height. The corresponding maximum dynamometer reading was 425 pounds per square foot, the center of the pressure plate being 1.2 feet above high-water level. The exposed face of the block was 11.4 square feet, and the least mean pressure per square foot which could have moved it when submerged is 132 pounds.

An embankment of riprap perpendicular to the direction of wave travel, and in water about 4.5 feet in depth at high water, was completely destroyed by waves, and the pieces washed shoreward an average distance of about 28 feet, and piled almost parallel to their former position and at a higher elevation on the sloping beach. The embankment was originally 5 feet in top width, with side slopes of 1 vertical to 3 horizontal, and was composed of pieces of limestone from 40 to 200 pounds in weight, carefully placed. The average weight of the stone per cubic foot was 135 pounds. The crest of the embankment was 0.5 feet above high-water level before it was displaced, and 1.5 feet above the same datum when washed up on the beach. A dynamometer set in the vicinity, with the center of its plate 1.7 feet above high-water level, gave on this occasion a maximum reading of 625 pounds per square foot.

Owing to the irregular shape of the riprap it is not possible in this case to make even an approximate computation of the force required to move the largest pieces, but that indicated by the dynamometer was evidently more than sufficient, as every piece of stone in the entire embankment was moved from its original position.

EFFECTS DUE TO WAVES ON THE GREAT LAKES.

Overturning cribs at Milwaukee, Wis., 1891 and 1893.—During a storm January 1, 1891, the crib described in detail in paragraph (18), Chapter VIII, was overturned by waves in water of a depth of 32 feet. The officer then in charge of the work, from known data, computed the submerged weight of the crib at 11.74 tons per linear foot of crib. Assuming normal wave impact, as the breakwater was parallel to the shore, and that the center of pressure was at the center of the exposed face, it would require a mean pres-

sure of about 1,100 pounds per square foot to overturn this crib. Although its exact position is not known, it is probable that the center of pressure in such a case is above the center of the exposed face, consequently the mean pressure which overturned the crib was probably somewhat less than 1,100 pounds per square foot. In overturning on the interior bottom edge the center of gravity would be raised 5.2 feet, hence it would require a theoretical maximum energy of 61.05 foot-tons per linear foot of crib to overturn it.

No wave measurements are on record for this locality, but the officer in charge estimated the maximum waves to be about 12 feet in height. From experience on Lake Superior it seems probable that the waves in this storm were at least 200 feet in length, and probably more.

Such a wave by equation (24) would give a possible maximum dynamometer pressure per square foot of 2,436 pounds, and would have a total theoretical energy of about 109.1 foot-tons per linear foot of wave, measured parallel to the crest.

At the same locality, during a storm April 19, 1893, two cribs, described in detail in paragraph (19), Chapter VIII, were overturned by waves.

Lieut. (now Capt.) C. H. McKinstry, U. S. Corps of Engineers, assistant to the officer in charge of the work, computed that, if the decking remained unbroken, one of these cribs would just be overturned by a force of 21,120 pounds per linear foot of crib, acting 11.26 feet below the top of the crib. The mean pressure therefore per square foot on the exposed face would be 797 pounds. If the decking were previously destroyed, permitting some of the stone ballast to be washed out and the waves to act against the upper part of the inner wall, less pressure per square foot would be required on the exposed face.

With decking intact, the minimum theoretical energy per linear foot of crib required to overturn it is 62.02 foot-tons. The maximum wave height is given as 13 feet. If the wave length be taken as 200 feet, as in the preceding case, such a wave would, from equation (24), give a possible maximum dynamometer pressure of 2,409 pounds per square foot, and would have a total energy of about 127 foot-tons per linear foot of wave crest.

In 1893–94 seven dynamometer readings were secured at Milwaukee Harbor, which were as follows:

Date.	Maximum dynamometer readings. (Pressure per square foot.)		Elevation of dynamometer plate above water surface.
	A.	B.	
	Pounds.	*Pounds.*	*Feet.*
April 19, 1893	540	....	1.0
February 12, 1894	200	[a]1,430+	6.5
April 8–9, 1894	300	[a]3,460+	2.5
May 18, 1894	316	1,970	2.5

[a] Spring compressed to full limit.

On February 12, 1894, both dynamometers were fastened on top of the superstructure of the breakwater, dynamometer A, with its face flush with the exposed face of the breakwater, and dynamometer B facing in the same direction, but set back near the harbor side of the breakwater. On the other dates the dynamometers were set, as just described, on top of a crib without superstructure. The top timber just below dynamometer A was cut away on a 45° bevel, so that the plate overhung the entire beveled edge. It will be observed that the readings of this dynamometer are relatively much smaller than those for a corresponding dynamometer on the breakwater at the upper entrance of the Portage Canals. This is probably due to the fact that the vertical face of the crib at Milwaukee deflected the water to such an extent as to cause it to strike the face of dynamometer A at a considerable angle, or it may possibly have resulted from back pressure on the dynamometer plate, caused by water striking the support to which the instrument was fastened. The reading given by dynamometer B on April 8–9 is considerably larger than any yet obtained on Lake Superior, and was probably due either to concentrated wave energy or to some floating solid body like ice or timber striking the pressure plate, as the reading is nearly double that on May 18, when the wind velocity was slightly greater, as was also the reading of dynamometer A.

Damage to breakwater at Buffalo, N. Y.—The injury to the breakwater at Buffalo, N. Y., described in paragraphs (20) and (21), Chapter VIII, was due neither to direct nor to

transmitted wave action, but was caused by falling water, which had been thrown to a great height by waves striking the vertical face of the breakwater.

Assuming that the mass of falling water corresponded to a uniform load suddenly applied, and that the unit stress of the timber was 6,000 pounds per square inch, we find that the minimum breaking load for the 12 by 12-inch ties described in paragraph 20 corresponds to a mean pressure of about 1,145 pounds per square foot over the entire area of 50 square feet of deck supported by each tie.

As many of these ties were broken during the unusually severe storms in 1899 and 1900, it would appear that the mean pressure per square foot over an area at least as large as 50 square feet must have been greater than 1,145 pounds.

Storms previous to the unusually severe ones of 1900 had shown the liability of the type of deck just described to damage from wave action, and the deck system constructed thereafter was strengthened by placing a center longitudinal timber between the deck ties and the next tier below. In subsequent construction the deck ties were spaced 3 feet 4 inches between centers, instead of 5 feet, as had been done previously.

Where the deck system was reinforced by the center longitudinal timber no ties were broken, but when the ties were spaced 5 feet between centers deck plank were broken into smaller pieces than on the original deck.

Under the assumption previously made, the minimum breaking load for ties spaced 5 feet from center to center, and reinforced by a center longitudinal, corresponds to a mean pressure of about 1,718 pounds per square foot over the entire area of 50 square feet. As none of these ties were broken, it seems evident that the mean pressure per square foot over an area as large as 50 square feet could not have been as much as 1,718 pounds per square foot.

It would therefore appear from the two preceding instances that for an area as large as 50 square feet the destructive force upon the decking at Buffalo, N. Y., corresponded to a pressure lying between 1,145 and 1,718 pounds per square foot.

In the case of the 3-inch deck plank, 8 inches in width, resting on ties spaced 5 feet between centers, each plank

forming a continuous beam, the minimum breaking load for the weakest panel is about 2,630 pounds per square foot over a minimum area of 6 square feet.

For the same decking resting on ties spaced 3 feet 4 inches between centers, each plank forming a continuous beam, the minimum breaking load for the weakest panel is about 8,100 pounds per square foot over an area of 3.8 square feet.

The officer in charge states that in the last case deck plank were rarely broken by waves, and that the few broken were probably weakened by knots.

The displaced concrete banquette section at Buffalo, N. Y., described in paragraph (22), Chapter VIII, consisted of a monolithic concrete banquette deck 14 feet in width, 36 feet in length, and ranging from 2 to 4 feet in thickness, which weighed about 111 tons and rested upon two rows of concrete blocks and an interior rubble filling between them. There were three blocks in each row, their dimensions being as follows: Width, 4.5 feet; length, 12 feet, and height 4 feet. All blocks were provided with sunken panels on top, to prevent the monolithic deck from sliding. In addition, the harbor-face blocks were provided with joggle channels at the ends, which were filled with concrete as soon as practicable after the blocks were placed. The combined weight of the six concrete blocks upon which the banquette deck rested was about 97 tons.

The displacement was probably caused by wave impact against the vertical lake face of the section, the stone in front of which had doubtless been previously washed out by waves down to the level of the bottom of the concrete blocks. The effective weight of the section was probably reduced by submergence to about 122 tons, and possibly still further reduced by upward pressure transmitted through the interstices of the timber and rubble bed upon which the section rested.

On the other hand, the displaced section was anchored to the concrete lake-face blocks by two rods of $1\frac{1}{4}$ inch iron, each about 18 feet in length. The rod nearest the end which had moved most was broken loose, and in both cases the hooks and rings to which they were fastened were bent. The core of rubble filling under the banquette deck and between the concrete blocks also opposed a resistance to the movement of the section. Experiments made at the United

States engineer office at Duluth, Minn., in 1902, by Mr. J. H. Darling, United States assistant engineer, showed that for a crib with pockets open at the bottom and filled with rubble of an average size of about 1.7 cubic feet and resting upon a mound of similar rubble, the coefficient of resistance against sliding was about 0.90. For the coefficient of friction of concrete blocks resting on a rubble base, the coefficient of friction is taken as 0.65.

Assuming a mean coefficient of friction for the whole of 0.75, that the entire section displaced was submerged at the time and that the effect of any upward pressure against the bottom was counterbalanced by the resistance afforded by the two anchor rods, we have for the least mean pressure capable of moving the section, acting against the exposed area of 252 square feet, about 726 pounds per square foot.

During the storm of September 12, 1900, a number of the lake-face concrete blocks described in detail in paragraph (23), Chapter VIII, were washed overboard, falling lakeward. Each block contained 9.45 cubic yards of concrete, and weighed when submerged nearly 23,000 pounds. The rear face presented an area of 36 square feet, and the block rested on a horizontal bed. Assuming a coefficient of friction of 0.65, and that the joggles were previously broken, or that a large number of the blocks were washed overboard simultaneously, we find that the least mean pressure per square foot on the rear face of a block which would have moved it is 415 pounds. The force, acting at the center of the rear face, which would be required to overturn the block would be 1,021 pounds per square foot. It is not likely, therefore, that the blocks were displaced by overturning.

The manhole covers which were displaced, as described in paragraph (24), Chapter VIII, could have been lifted by a pneumatic pressure of 75 pounds per square foot.

No dynamometer measurements and no measurements of wave dimensions have been made at Buffalo Harbor, but the officer in charge of the works of improvement there states that the maximum storm waves are believed by the assistant engineers not to exceed 100 feet in length and 10 feet in height. From equation (24) such a wave in water 30 feet in depth would give a possible maximum dynamometer reading of about 1,675 pounds per square foot.

Its total theoretical energy per foot of wave crest would be about 36.9 foot-tons, and if it were possible for all of this energy to be so expended it would be sufficient to throw a cubic yard of water to a height of about 44 feet.

Movement of stone at Black Rock, Lake Superior.—In the case of the third mass of stone moved at Black Rock, Presque Isle Park, Marquette, Mich., described in detail in paragraph (25), Chapter VIII, the exposed face was fairly regular in outline and its area was about 12 square feet.

Supposing the entire mass to be submerged, and assuming, on account of the upward slope and character of the rock surface, a coefficient of friction of 0.85, we find that the least mean pressure per square foot, which would move the mass is 634 pounds—a remarkable result, when the distance from the lake, 125 feet, and the height of the stone, 15 feet above low-water datum, are considered.

On December 3, 1902, a spring dynamometer, located about 110 feet away and 8 feet from the water's edge, with its plate 3 feet above low-water datum, gave a reading of 2,055 pounds per square foot in a storm much less severe than several which have occurred at the same locality during the past ten years.

Damage by waves at Marquette, Mich.—At Marquette Harbor, Mich., during a storm in June, 1899, the concrete blocks described in paragraph (28), Chapter VIII, were washed overboard, falling toward the oncoming waves. These blocks when submerged weighed 5,300 pounds each and rested on timber supports. Assuming a coefficient of friction of 0.6, it is found that the least mean pressure per square foot which would move them is 318 pounds. It should be stated that the blocks were prevented from moving in the direction of wave impact by a part of the timber superstructure of the breakwater immediately in their rear. The waves striking this caused a back pressure on the rear of the blocks.

In the same harbor during a very severe storm in December, 1895, the wooden ties and decking described in paragraph (30), Chapter VIII, were broken by water falling upon the deck of the breakwater, which was about 5 feet above low-water datum. Assuming, as in the similar case of the breakwater at Buffalo, N. Y., that the falling water represented a uniform load suddenly applied, and that the unit stress of the

timber in this case was 6,000 pounds per square inch, we find for the least breaking load per square foot for the ties 993 pounds, and for the decking 996 pounds.

There have been no storms of any consequence from the single direction of exposure since a dynamometer was installed on this breakwater in 1901, but the maximum reading at Black Rock, 2½ miles distant, during this period was 2,055 pounds per square foot.

Damage to west breakwater, upper entrance, Portage Canals, Mich.—In September, 1901, during an unusually severe storm on Lake Superior, the damage described in paragraph (29), Chapter VIII, occurred to the west breakwater at the upper entrance of the Portage Canals, Mich. New 6 by 12-inch white-pine timbers, 16 feet in length, forming the sloping face, were loosened for 8 feet from their upper ends and broken off at mid length by a reverse pressure acting from within the superstructure itself. Considering the timber as a beam 7.5 feet in length, fastened at one end, to which a uniform load is suddenly applied, we find the least breaking load per square foot to be 640 pounds. A spring dynamometer but a few feet distant, and with its plate set about 3.5 feet above the center of the broken sections of the timbers, gave a maximum reading of 2,525 pounds per square foot on this occasion.

The top wall-timber of the inner face of the superstructure of the breakwater at the upper entrance, Portage Canals, Lake Superior, is a 12 by 12 inch white-pine timber 18 feet in length, held in place by six round iron bolts, each 35 inches long, driven through the center of the upper face of the timber into the solid wall of similar timbers beneath. The top of the timber is flush with the upper surface of the deck of the breakwater, which consists of 6 by 12 inch decking, spaced 1.3 inches apart, and laid horizontally at right angles to the axis of the breakwater. The ends of the deck plank are separated from the lake face of the top wall-timber by a narrow space varying from 0.1 to 0.3 of an inch. Direct wave impact against the exposed face of this wall-timber, therefore, can only take place through the 1.3-inch openings between deck plank.

During the storm of September 16, 1901, all upper wall timbers on the west breakwater for a distance of more than a thousand feet were displaced by revolving on the lower inner

edge, bending and partly drawing the iron bolts through the top timbers, so that a few of these timbers revolved as much as 90° from their original positions, but most of them for only a few degrees. No timbers were split, nor were any torn entirely loose from the bolts.

Assuming, as has been determined by experiment by Noble and Gilbert, that the force required to start one of the bolts to draw through the 12-inch top timber is about 6,170 pounds, and that the force required to bend the bolt, if suddenly applied, is 331 pounds, we find that the minimum mean pressure per square foot which would have produced the effect described is 2,167 pounds.

During this storm a dynamometer set on top of the deck of this breakwater 18 feet in front of the displaced timber, and with the center of its plate 1.25 feet above the center of the exposed face of the timber, gave a maximum reading of 2,525 pounds per square foot.

The minimum mean pressure per square foot against the inner face required to move the sandstone blocks described in paragraph (l), Chapter IX, assuming a coefficient of friction of 0.75, is 354 pounds.

In this case the movement of the blocks was in a direction opposite to that of wave travel, and was probably due to reflected wave action. The centers of the displaced blocks were about 17.5 feet below the water surface at the time. No dynamometers were in place then, but the maximum reading since, on dynamometers set about 7.5 feet above the water surface, for storms believed to be of about equal intensity, have varied between 1,800 and 2,200 pounds per square foot.

Movement of masses of concrete and stone at Duluth, Minn.—To overturn one of the concrete blocks described in paragraph (31), Chapter VIII, as was done by waves at the Duluth Canal in October, 1899, would have required a minimum mean pressure per square foot of 714 pounds with the block submerged. To have moved it along on the deck of the crib a mean pressure of 257 pounds per square foot would have been required.

No dynamometer was in position at that time, but one was afterwards set at the same elevation and within a few feet of

the same spot for two seasons. Its maximum reading during this period was 1,630 pounds per square foot.

To have raised the mass of trap rock described in paragraph (32), Chapter VIII, if submerged, would have required a sustained pressure of at least 400 pounds per square foot during the entire time that the block was lifted vertically more than 5 feet.

For more ready comparison, the results of the preceding computations have been condensed in the following table:

TABLE XXV.—*Area exposed to wave action, minimum force required to produce observed wave effects, and maximum observed or computed dynamometer pressures, in the instances cited, in the present chapter, of damage caused by wave action.*

Locality.	Area exposed to wave action.	Minimum pressure per square foot required to produce observed results.	Maximum dynamometer pressure per square foot.		Remarks.
			Observed near same locality.	Computed from equation (24) f = 2.0.	
	Sq. ft.	*Pounds.*	*Pounds.*	*Pounds.*	
Wick, Scotland	546	2,015	....	6,340	Wave length estimated.
Do	105	675	....	6,340	Center of blocks 23.5 feet below high water.
North Beach, Fla	60	200	633	....	
Do	12.5	200	533	....	
Do	11.4	132	425	....	
Milwaukee, Wis	2,250	1,100	....	2,436	Wave dimensions estimated.
Do	1,325	797	540	2,409	Do.
Buffalo, N. Y	50	1,145	....	1,675	Do.
Do	6	2,630	....	1,675	Do.
Do	252	726	....	1,675	Do.
Do	36	415	....	1,675	Do.
Do	7.1	75	....	1,675	Do.
Black Rock, Marquette, Mich.	12	634	2,055	....	Dynamometer readings at Black Rock not on same date.
Breakwater, Marquette, Mich.	10	318	2,055	....	Do.
Do	9	996	2,055	....	Do.
Do	72	993	2,055	....	Do.
Upper entrance Portage Canals.	7.5	640	2,525	....	
Do	18	2,167	2,525	....	
Do	10	354	1,800-2,200	....	Displaced stone 17.5 feet below water surface.
Duluth, Minn	18.8	714	1,630	....	Dynamometer reading not on same date.
Do	15	400	2,370	....	Do.

In the preceding table there are but five instances where corresponding observations were secured from dynamometers set but a few feet away from the object displaced, or injured. These instances are the third, fourth, fifth, seventeenth, and eighteenth, cited in the table, and it will be seen that in these cases the minimum computed force capable of producing the observed effect varies from 26 to 86 per cent of the corresponding observed maximum dynamometer reading. The surface exposed to wave action varies in the five cases mentioned from 7.5 to 60. square feet.

It is interesting to note that the minimum computed mean pressure required to force back the top wall-timber at the west breakwater, upper entrance of Portage Canals, Michigan, 2,167 pounds per square foot, is slightly greater than that (2,015 pounds per square foot) required to move the immense mass of concrete at Wick. In the former case, however, the exposed face was but 18 square feet in area, while in the latter it was about 546 square feet, the vertical dimension of the exposed area in the former case being but 1 foot, and in the latter 21 feet.

In the ordinary case of water thrown by wave action to a considerable height and falling upon a surface like the deck of a breakwater, equation (25) should apply. That it does apply as closely as might be expected has apparently been shown by the experiments described in Chapter XII.

When a shallow-water wave strikes a high vertical wall a part, at least, of its energy is exerted in projecting a mass of water upward and parallel to the face of the wall. The thickness of the projected mass of water, measured perpendicularly to the highest part of the face of the wall is usually not great, and the total volume of water projected is considerably less than that composing the wave which produced the effect, or, in other words, a part, at least, of the wave energy is concentrated upon a smaller volume of water than in the original wave, and is therefore capable of producing greater effects upon exposed areas of limited dimensions.

Other things being equal, the greater the mass of water projected upward, the less will be the height which it will attain. When the height of projection is considerable the continuity of the fluid particles will not be maintained, and the falling water will be broken up and mixed with air. Such a

mass of water falling upon the deck of a breakwater from a known height can not give a pressure as great as that indicated by equation (25) when f' is equal to 1.0, which is the maximum *possible* value of f', and *not* the *probable* value, unless the height of projection is not excessive, and the falling mass of solid water is large enough to cover the entire area considered, and its vertical thickness sufficient to develop the pressure due to the velocity of impact.

Assuming that these three conditions existed in the cases of the ties and decking broken by falling water at Buffalo, N. Y., and Marquette, Mich., and making $f'=1.0$, we find that to break the ties at Buffalo, a solid mass of water at least 50 feet in area would have to fall from a height of not less than 18.4 feet, while to break the deck plank at the same place the solid mass of water would have to be not less than 9 feet in area, and fall from a height of not less than 42.2 feet.

To break the ties at Marquette, Mich., the area of the mass of water would have to be not less than 72 square feet, falling from a height not less than 16 feet, while to break the deck plank at the same place these quantities would be 9 square feet and 16 feet, respectively.

It will be remembered that at Buffalo, N. Y., the officer in charge of the work states that during the gale which caused the damage described, "the waves dashing against the vertical walls of the structure rose to a great height above it, variously estimated at from 75 to 125 feet, etc."

In view of this statement the values just deduced for the minimum height of fall appear reasonable in all four cases considered.

That, within certain limits, the smaller the area the greater will be the force exerted upon it by the falling of water thrown up by waves was clearly shown at Buffalo, where at no place was a force so great as 1,718 pounds per square foot exerted over an area so great as 50 square feet, while in numerous places a force of more than 2,630 pounds per square foot was developed over an area of 6 square feet.

The reason for this is that a falling mass of water may be "broken" as regards a large area, but solid with respect to a smaller area. For this reason the individual deck plank should, at sites exposed to wave action, always be relatively stronger than the ties upon which they are supported.

MOVEMENT OF MATERIAL ON THE BOTTOM DUE TO WAVE ACTION.

The movement of dredged material described in paragraph (*r*), Chapter IX, was probably due principally to breaking waves combined with undertow, until the depth was increased by their action from 19.4 to about 28 feet, ordinary storm waves seldom breaking at this locality in water deeper than 28 feet. During the past two seasons waves longer than 200 feet have been observed at the Duluth Canal on three occasions only, while waves 200 feet in length have been observed on six other occasions. Waves exceeding 14 feet in height have been observed on five occasions, and waves just 14 feet in height were noted on three other occasions. It may then be assumed that the ordinary storm waves at this locality do not often exceed 200 feet in length and 14 feet in height.

Assuming these dimensions, and that two-thirds of the wave height is above still-water level, the corresponding theoretical wave velocity for a depth of 28 feet is 27.5 feet per second and the wave period 7.3 seconds. The orbit of a particle on the bottom is a straight line 12.7 feet in length over which the particle passes twice in 7.3 seconds. Its mean velocity is therefore 3.5 feet per second and its maximum velocity, at the mid point of the orbit, is from equation (21) 5.5 feet per second.

When the depth is 38.4 feet, as was the case when soundings were taken in February and April, 1902, the linear orbit of a bottom particle for such a wave is 8.4 feet, its mean orbital velocity 2.5 feet per second, and its maximum orbital velocity 3.9 feet per second.

These velocities appear ample to displace material of the character here encountered, and it is probable that at the next annual survey an increase of depth will be found at this dumping ground.

The deposit of sand on the piers at Grand Marais, Mich., and Duluth, Minn., as described in paragraph (*s*), Chapter IX, may be attributed to waves which break in the vicinity and stir up the bottom to such an extent that the water is discolored for some distance around.

At the upper entrance of the Portage Canal and at Marquette, owing to greater depth, the sand is believed to be

deposited by waves which are unbroken until they strike the breakwater. The larger storm waves at these localities probably have about the same dimensions as those just described at Duluth. Making this assumption, we find that in the former case, for a depth of 30 feet, the wave period is 7.1 seconds, the linear orbit of a particle on the bottom 11.7 feet, the mean velocity of a bottom particle 3.3 feet per second, and the maximum velocity of the same particle 5.2 feet per second.

At Marquette, Mich., where the depth is 36 feet, the wave period would be 6.8 seconds, the linear bottom orbit 9.2 feet, the mean orbital velocity of a bottom particle 2.7 feet per second, and its maximum orbital velocity 4.3 feet per second.

It is to be regretted that there are no measured wave dimensions available for use in connection with the very interesting attempt, described in paragraph (*t*), Chapter IX, to discover the limit to which wave action extends at Milwaukee, Wis.

Assuming, for reasons previously stated in this chapter, that the larger storm waves at this locality are 13 feet in height and 200 feet in length, that two-thirds of the wave height is above still-water level, and deducing the theoretical mean and maximum orbital velocities of bottom particles at the depth of 44 feet, which is stated to be the limit of very decided wave action, we have for the former 1.9 feet per second and for the latter 3.1 feet per second.

In a depth of 60 feet it is stated that there was no indication of any disturbance of the dredged material. Assuming the same wave dimensions as before, we find the mean and maximum bottom orbital velocities for a depth of 60 feet to be 1.2 and 1.9 feet per second, respectively.

From paragraph (*u*) Chapter IX, it is seen that the mud or clay bottom of the lake in the vicinity of the Duluth Canal changes to sand when the depth is less than 55 to 60 feet. Taking the latter depth as marking the limit of appreciable wave action on the bottom, and assuming a wave 14 feet in height and 200 feet in length, with two-thirds of its height above still-water level, we find the wave period to be 6.4 seconds, the linear orbit of a particle on the bottom to be 4 feet, the mean orbital velocity of the corresponding particle to be 1.3 feet per second, and its maximum orbital velocity 2 feet per second.

In cases like those last discussed, it is impossible to say just

what orbital velocities on the bottom are sufficient to move different classes of materials, for the theoretical velocity of the particle in its linear orbit is continuously changing. During a single wave period of six or eight seconds, this velocity twice becomes zero, and twice attains its maximum value; in the latter case when the highest point of the crest and the lowest point of the hollow are directly overhead, and in the former case when the mid height of the posterior and anterior slopes of the wave are passing.

It would, therefore, be useless to attempt to compare these velocities with such current velocities as have been determined by experiment to be capable of moving materials of various kinds and dimensions.

In this connection it should be stated that so far as the writer's experience is concerned, the effects of current velocities in laboratory experiments are considerably greater than those produced by equal velocities on natural bottoms. In the one case the material acted upon is usually carefully assorted, freshly in place and uncompacted by previous disturbing influences; while in the other case the particles are of various sizes and probably interlocked and compacted, since by long exposure to the same forces, the less stable particles would previously have been displaced and moved to more stable positions.

While nothing absolutely *conclusive* has been shown by the comparison of observed wave effects with results indicated by theory or by dynamometer measurements, yet, upon the whole, the comparison must be regarded as satisfactory, inasmuch as in *not a single instance* was the observed effect inconsistent with theory or with the nearest available dynamometer reading.

CHAPTER XIV.

Conditions modifying the effects of wave action. Character of the site. Form of structure. Angle at which waves impinge.

The extent to which a structure will be exposed to wave action depends upon the area and depth of the adjoining body of water, the frequency and violence of storms; the "fetch;" the governing depth and slope of the bottom in the vicinity; the fluctuations of the water level; the form and position of the exposed surface, and the angle at which waves impinge upon it.

It is evident without demonstration that huge waves like those encountered on the Atlantic or the Pacific can not be generated upon bodies of water of restricted area, however violent the action of the wind may be. With equal "fetch" and wind velocity the length and height of waves upon a comparatively shallow lake, like Lake Erie, will be considerably less than upon one like Lake Superior, where the depths are very much greater. The effect of wave action is often modified to a marked degree by the depth in front of the exposed face of the structure, for, as has been shown in a previous chapter, no wave can exist in a given depth of water of a height greater than that depth, and ordinarily a wave breaks when its height is much less than the depth of the water in which it is traveling. If, therefore, the depth in front of a structure is less than that in which storm waves ordinarily break, it is evident that the work can not be assailed by waves of the largest size. For the same reason it follows that in many cases against such structures the more violent storms do not produce the greatest destructive effect, for an outlying shoal or submerged ledge will often trip up the larger waves of such a storm, causing them to break before reaching the structure, but will permit the smaller waves of a less violent storm to pass unbroken, and exert their energy against the structure itself. The protective effect of shoal water against wave action is often a most important modify-

ing influence, and one which does not always receive the consideration it merits. The slope and configuration of the bottom in the immediate vicinity of a structure also exercise an important influence upon the action of waves against the work, tending in some cases to concentrate wave action, and in others to reduce it.

Breakwaters, as usually constructed, are of three general types: (1) In which the exposed face is vertical; (2) partly vertical and partly inclined; and (3) an inclined surface. The relative advantages of the three types have been discussed at great length, and each finds earnest and able advocates. Those favoring the vertical wall sometimes claim that the action of waves against it is purely oscillatory, and that the only force it encounters is the hydrostatic pressure due to the height of the waves which rise up against it without breaking. This may be true in water of great depth, but breakwaters are usually located in water of but moderate depth, and many of them at places where waves would break in severe storms from lack of sufficient depth, even if no breakwater existed at the spot. In such cases the vertical-face breakwater is at times subjected to very heavy wave impact, although the greater number of the waves which would attack it are reflected, and, meeting those advancing, produce by wave interference a chaotic movement of the water for a considerable distance out from the work. Some of the oncoming waves are neutralized and a smaller number augmented, so that the vertical wall occasionally receives an impact greater than would be the case if no reflection occurred. For example, at Duluth, Minn., at the head of Lake Superior, a spring dynamometer with a disk 1 square foot in area was fastened to a vertical concrete wall with the disk parallel to the same. The center of the disk was 2.5 feet above the lake level and 6.5 feet below the top of the wall. The depth of water immediately in front of the dynamometer was about 6 feet, increasing lakeward. During a northeast storm on October 23, 1902, this dynamometer indicated a pressure of 2,430 pounds per square foot. As explained in Chapter XII, the spring dynamometers here used can not measure static pressure; therefore the recorded pressure, which is an unusually large one for waves in water but 6 feet in depth, must have been due to dynamic action alone.

In many cases a combination of sloping and vertical faces is used in the same breakwater, as where a rubble embankment is surmounted by a superstructure with a vertical or a sloping face. The superstructure of a vertical-face breakwater of this type is sometimes subjected to heavier impact than that of one of the first or third types, owing to the fact that the waves run up or break upon the slope of the rubble embankment and act with concentrated effect upon the superstructure.

The function of the sloping-face superstructure is to cause the mass of water impinging against it to move up an inclined surface, thus expending a part of its energy in the elevation of a mass of water instead of in destructive action upon the breakwater itself. Instances of this action are seen in the photographs on pages 214 and 215. By some writers it has been claimed that the mass of water which passes over the sloping face of a breakwater and falls in rear causes swells or waves of sufficient size to damage or incommode shipping. This may be the case where the top of the breakwater is low, but is not true where it is given a reasonable height and width. For example, the sloping-face breakwater shown in the photograph on page 223, which is located at the Upper Entrance of the Portage Canals, Lake Superior, is 7 feet above ordinary storm level of the lake and is 19 feet in width on top. The inclined face has a slope of 1 on 1, which extends about 4 feet below storm level of the lake. The rear wall is vertical, as is also the front wall below the sloping face. During severe storms waves from 12 to 15 feet in height assail this breakwater, and at times immense masses of water pass over it, falling in rear at some distance from the breakwater, but producing no waves or swells of appreciable size. The action of this breakwater in reflecting waves is very marked. A perfect chaos of waves, due to the meeting of advancing and reflected waves, extends lakeward from 1,000 to 1,200 feet. Yet in spite of this reflection waves frequently break from insufficient depth just in front of the breakwater and curl over upon it, producing severe normal impact upon the inclined face. On page 214 is shown a photograph of the breakwater at Marquette, Mich., on the south shore of Lake Superior during a storm. The concrete superstructure of this breakwater consists, on the exposed face, of two slopes, each 1 on 1, connected by a horizontal banquette 6 feet in width.

WAVE STRIKING CONCRETE SUPERSTRUCTURE OF BREAKWATER AT MARQUETTE, MICH.

WAVE STRIKING THE SLOPING FACE OF THE ROCK-FILLED TIMBER BREAKWATER AT PRESQUE ISLE, MARQUETTE HARBOR, MICH.

The width at the top is 4½ feet, and the height above storm level of the lake is about 9 feet. The rear face of the superstructure is vertical, as are both faces of the substructure. An inspection of the photograph shows that the directive effect is given almost entirely by the lower slope, so that there is actually a vacant space in the angle formed by the junction of the upper slope with the banquette. With this breakwater, as with the preceding one, waves are reflected to a considerable distance out from the breakwater, and large quantities of water pass over the top of the structure without creating undue disturbance in the harbor. This breakwater possesses ample stability, yet it undoubtedly receives at times more severe impact from waves breaking just in front of it, and falling over in the angle between the banquette and the foot of the upper slope, than would be the case if there were but a single continuous slope.

The photograph on page 215 shows the action of waves on the detached breakwater at Presque Isle, Marquette Harbor, Mich. The exposed face of this breakwater above the level of the water is a continuous slope of 1 on 2.4. The width of the breakwater is 24 feet, its height above storm level 9 feet, and its top width 1 foot. Both the front and rear faces are vertical below their junctions with the inclined face. Although the sloping face of this breakwater is unusually inclined it reflects waves out to a distance of several hundred feet, but this action is less marked than in the two cases previously described. As can be seen from the photograph, the mass of water passing over the top of the breakwater strikes the surface of the water at some distance from the rear wall, but produces no marked wave action.

The main breakwaters at the upper entrance of the Portage Canals, Lake Superior, are connected with the shore by short extensions of the following cross section above low-water datum, which datum is about 1 foot below the ordinary level during storms. Width at low-water datum, 20 feet. Height above same datum, 4.33. Top width, 1 foot. Front and rear slopes similar and each 1 on 2.2. Below low-water datum both front and rear faces are vertical.

During severe storms but little wave reflection is caused by these portions of the breakwaters, and solid masses of water pass over, creating considerable disturbance in rear, which,

owing to the location and to the short length of the sections in question, is not harmful, but would prove very serious if the whole breakwater had the same cross section.

From what precedes and from other evidence available, it would appear (*a*) that a sloping-face breakwater with vertical back wall, located in water less than 30 feet in depth, will afford sufficient protection to the area in rear, both against direct wave action and against waves generated by the water passing over it, provided its height above water during storms is not less than two-thirds the height of the greatest waves which strike it; (*b*) that unless the height and top width of the breakwater are unduly great so much water will pass over the top during severe storms that it would wash completely over the decks of vessels attempting to lie alongside of the breakwater; (*c*) that both the vertical and sloping face breakwaters will reflect waves, provided the height is sufficient, even although the angle of slope with the horizontal is as small as 23°.

If a surface exposed to wave action is of considerable extent it may occasionally experience severe impact, no matter what form it may have. As an instance of this, attention is invited to the photograph on page 216, which shows waves about 16 feet in height striking the outer end of the south pier of the Duluth Ship Canal during a storm in 1901. The end of the pier is triangular in plan, the two inclined faces, which are each 22 feet long, extend 18 feet above low-water datum, making an angle of 45° with the axis of the pier. Each face has a batter of 1 inch in 1 foot. The outer end of this pier is shown in the photograph on page 156. The substructure conforms in plan to that of the concrete superstructure. It would seem that a mass of water striking the angular end of the pier would be divided and thrown partly upward, but principally to one side, somewhat as water is displaced by the bow of a vessel. This is usually what occurs, yet the photograph shows that in this case, although the waves are traveling parallel to the axis of the pier, a volume of water has been thrown to a height of about 40 feet, as a result of a wave striking the end of the pierhead.

If the exposed surface is small, yielding, and of circular cross section, it sometimes appears to encounter but little wave energy. For example, parallel to the south pier of the

STORM WAVES IN THE DULUTH CANAL, 1901.

Duluth Canal, and about 300 feet from it, four piles were driven in a row in depths of 21, 16, 11, and 6 feet, respectively, for the purpose of measuring wave heights, velocities, etc. These piles were of pine, about 1 foot in diameter at the larger end, and projected about 10 feet above the water. Although waves broke around and against them in several storms during the season of 1902, they suffered no damage whatever from this source, although dynamometers in the vicinity have recorded pressures of over 2,000 pounds per square foot. Each pile presented several square feet to the action of the waves, and in the case of the one in the deepest water a horizontal force of but three or four thousand pounds, acting a little above the water level, should have been sufficient to break it off at the bottom.

That this did not occur is possibly due in part to the circular cross section of the pile, and in part to its yielding by bending upon wave impact.

Oblique impact of waves.—The angle with which the waves will impinge upon a pier or breakwater is of great importance in determining the extent to which it may be injured by wave action. If we denote by φ the angle between the vertical face of a structure and the direction of wave travel, assumed as horizontal, by F the wave force per unit of surface normal to this direction, and by F′ the force per unit of surface acting perpendicular to the face of the structure, we have—

$$F' = F \sin^2 \varphi.$$

This theoretical relation must not be relied upon too confidently, as in many cases it does not embrace all action exerted by waves. For example, if $\varphi = o$; that is, if the waves are running parallel to the pier, the before-mentioned expression would show that no wave force is being exerted against it, whereas both observation and experiment show that there is often a very considerable static pressure due to the height of the waves running along the side of the pier. The reason of this is that the water is generally deeper on the side of the pier next to the channel than it is on the opposite side, and consequently waves travel faster on that side of the pier than on the other. As a result of this, opposite phases of wave action are opposed at different points along the pier—for instance, at one point a wave crest on the inside of the pier

may be opposite a wave hollow on the outside, while at another point the reverse condition exists. This action takes place during every storm at the Duluth Canal. The piers protecting the entrance consist of a substructure of rock-filled timber cribs upon a pile foundation. The cribs are each 100 feet in length, 24 feet in width, and 21 feet in height, and are surmounted by monolithic concrete blocks of the form shown in the photograph on page 65, which are each 10 feet in length and weigh 85.6 tons. When waves 12 feet in height pass along the sides of the pier the top surfaces of these blocks have a total movement of about one-tenth of an inch in a direction perpendicular to that of the piers. This movement is doubtless due to the compressibility and elasticity of the timber substructure, and results in no permanent displacement of the concrete superstructure.

When the crests of the waves are considerably higher than the deck of the pier, and especially when the pier is provided with parapet walls of less height than the wave crests, the deck may receive severe impact from the mass of water tumbling upon it as the wave passes along the pier.

If a pier, or breakwater, has a reentrant angle, or forms an angle with the shore into which oncoming waves are crowded so that the space available for further travel becomes narrower and narrower, the destructive effect of the waves will be greatly increased, owing to the concentration of their energy within more confined limits.

Somewhat analagous to this is the action of a breakwater, against which waves impinge obliquely, in causing them to accumulate size and energy as they travel along its front.

This action was particularly marked in the case of the west breakwater at Oswego, N. Y., on the south shore of Lake Ontario, which was over a mile in length. During westerly gales the waves coming from that direction struck this breakwater at a small angle, and were in part reflected and in part crowded against the breakwater, accumulating size as they traveled, until on reaching the east breakwater, they passed in solid volume over the tops of the snubbing posts 15 feet above the water level.

CHAPTER XV.

Action of waves in eroding and building up exposed shores, and in subjecting structures in northerly latitudes to strains from ice.

The action of waves in forming bars at the mouths of streams and in eroding and depositing material alongshore are phenomena of such common occurrence that only a brief description of them will be necessary.

On a shore exposed to wave action the tendency of storms is to form a continuous beach. The material composing the bottom is stirred up at small depths by waves, and when the latter strike the shore at an angle, a littoral current is produced, which carries sand and gravel with it alongshore and deposits this material in any opening in the beach where there is not enough current to carry it away. In addition to the material carried by this littoral current, each wave running up the beach at an angle takes with it some material in suspension, and receding in a direction making a contrary angle, carries back with it a certain amount of material which is cast upon the beach as before, the result being that in a prolonged storm a large amount of material is moved along the beach itself by successive waves in addition to that transported by the littoral current. When the mouth of a river or other outflowing stream is encountered, the material carried along the beach by waves and by the littoral current comes in contact with the outflowing water, which in many cases is itself charged with sediment, and is carried seaward and then deposited, forming a bar obstructing the entrance. On such bars there exists a continual struggle between the waves seeking to carry the material which composes them inshore and the outgoing current striving to push this material farther and farther from shore.

The conditions under which such bars can be formed are stated by Mr. David Stevenson as follows:

First. The presence of sand or shingle, or other easily moved material.

Second. Water of a depth so limited that the waves during storms may act on the bottom.

Third. Such an exposure as to allow waves being generated of sufficient size to act on the submerged material.

The normal condition of a sandy shore, or of a wave-formed bar, represents a condition of equilibrium of the various opposing forces. This condition is ordinarily so delicate as to be readily disturbed by certain classes of artificial works. For example, if a groin or a pier be constructed out from a sandy shore which is exposed to wave action, there will usually result an accretion of the shore on one side and erosion on the other, due to the fact that material ordinarily traveling along the beach during storms is caught and retained against one side of the pier, cutting off the normal supply of material from the other side upon which wave action continues, material being removed as before and none moving on to take its place.

The distance to which material is moved alongshore under the action of waves is in some instances surprisingly great. Thus, for example, siliceous sand predominates on the beach along the east coast of Florida to the extreme south end of the peninsula, although there is no siliceous rock adjacent to the coast within a thousand miles, from which this sand could be formed.

Between 1791 and 1887, the shore line of North Beach, Florida, between groins 1 and 2 (see Pl. XI), had receded 1,350 feet, while that of St. Anastasia Island, at a point about 2 miles south of the inlet, had receded 2,700 feet. The more rapid recession in the latter case is due to the fact that the sand traveling down North Beach presses the main channel close against the island, with the result that the slope of the bottom near shore is unusually steep, and consequently the backwash of the waves and the resulting undertow carry the eroded material into this channel and the current transports it away. As the beach to the north is cut off by the inlet, there is no moving material to supply its place.

As an instance of the extent to which sand traveling along the beach under wave action is caught and retained by a pier projecting from shore, may be mentioned the experience at Superior Entry, Wisconsin, at the head of Lake Superior.

The direction of the shore line at this locality is northwest to southeast, and the beach is flat and sandy, the travel of sand being from southeast to northwest. Parallel piers for the protection of the entrance were commenced in 1869. Between that date and the spring of 1902 about 725,000 cubic

yards of sand had been caught and retained by the south pier, causing the shore line at the pier to advance 1,000 feet lakeward. This movement of the shore line extended along the beach for a distance of 6,000 feet southeast of the pier.

At Grand Marais, Mich., on the south shore of Lake Superior, the beach is flat and sandy, and the shore line trends about S. 72° W., the travel of sand along the shore being toward the east. Parallel piers for the protection of the entrance were commenced in 1883. Between that date and July, 1902, about 1,260,000 cubic yards of sand had been caught and retained by the west pier, causing the shore line at the pier to advance 700 feet lakeward. The movement of the shore line extends up the beach for a distance of 3,300 feet.

Waves of appreciable height always break upon approaching shore, and if the depth near shore decreases very gradually and uniformly there will usually be several distinct lines of breakers during storms. If the slope of the bottom near shore is very abrupt there may be but a single line of breakers, or the waves may break against the shore itself. Each breaker throws a mass of water toward or on shore, tending to raise the water level in that vicinity. The water thus thrown shoreward seeks to flow back, but on shallow coasts is opposed by one or more lines of breakers tending to force it shoreward again.

On unusually flat, sandy shores this water escapes seaward by flowing parallel to the beach, well inside of the outer line of breakers, eroding a channel which is sometimes 200 feet in width and 2 to 8 feet in depth, the dimensions depending upon the character of the bottom and the violence of wave action at the locality. On straight, continuous coasts these channels sometimes continue parallel to the shore line for several thousand feet, and in all cases until the volume and velocity of the water which they carry is sufficient to overcome the opposition of the breakers, when they turn seaward and discharge their contents through the lines of breakers. These channels, or "sloughs," as they are locally called, are formed on the straight, flat, sand beaches of the east coast of Florida during every severe northeast storm, and gradually fill up during quiet weather. They are especially marked in the vicinity of inlets or the mouths of rivers, where the shore line curves

inward, enabling the channels to discharge their contents without opposing the breakers directly.

At North Beach, Florida, the writer observed and measured these storm-formed "sloughs" for two years, during the construction of groins planned to stop the erosion of the North Beach, to which this phase of wave action had largely contributed. A sketch of the locality is shown on Pl. XI. Before the construction of the groins the "slough" usually commenced about at *M* and extended parallel to the low-water line, to some point between groins 2 and 3, where it emptied into the inlet. The water of the inlet at groin 3 always flowed northwest, owing to an eddy formed there during ebb tide.

The groins were commenced during the winter of 1889–90, and completed April 14, 1890. A survey made on June 23, 1891, showed that the erosion of the point had been completely checked, and that over 140,000 cubic yards of sand had been collected and retained by the groins. This represented but a small proportion of the sand actually moving along the beach during this period, as for several months prior to the survey the sand-retaining capacity of the groins had been reached; the the low-water line had moved seaward beyond the extremities of the groins; the formation of the "sloughs," just outside of the low-water line, took place as before, and the travel of the sand from north to south had again been resumed. The same action of the waves in forming these narrow "pockets," or "sloughs," parallel to shore occurs to a marked degree at Grand Marais, Mich., where the bottom is flat and sandy, in a depth of from 4 to 12 feet, and to a less extent north of Superior Entry, Wisconsin, where somewhat the same conditions prevail. When it is desired to check the erosion of a shore along which sand travels, by the construction of groins, that one which is to be located farthest down the beach, with respect to the travel of sand, should be constructed first; for if one of the upper ones be constructed first it will cut off for a time the supply of sand which would otherwise pass along the beach below it, and consequently a lower groin, unless at a considerable distance from it, could perform no useful function in catching and retaining sand until the upper groin had caught most of the amount which it was capable of retaining.

When piers or jetties are to be built out from such a shore, a careful study of the sand movement should, whenever possi-

ble, be made in advance, for if this movement is excessive it will, owing to the advance of the foreshore, necessitate their extension to deeper water within a comparatively short period.

When the bottom in the vicinity of the shore is not sandy, and has not an unusually flat slope, the water cast shoreward by breaking storm waves recedes directly seaward, overcoming the opposition of the breakers. In order to do this the returning water naturally seeks the line of least resistance.

As has been shown previously, the intensity of wave action decreases rapidly toward the bottom; consequently the receding waters follow the bottom as closely as possible, forming the "undertow" familiar to residents of wave-exposed coasts. Material stirred up by waves tends to sink toward the bottom, and is often caught by the "undertow" and carried for a considerable distance in a direction opposite to that of wave travel. One instance of this is seen in the removal lakeward of dredged material from the submerged dumping ground in Lake Superior, near the Duluth Canal, described in Chapter IX. Shore erosion is in many cases greatly augmented by the action of the undertow, bluffs being undermined by waves and the material taken offshore by the undertow and subjected to the influence of littoral currents, by which it is transported away.

It must not be supposed that it is only upon sandy shores that wave action affects the configuration of the shore line. On the contrary, there exists scarcely a coast the shore line of which is not being gradually changed by the action of the waves. On some, bluffs are washed down; on others, rocky cliffs slowly undermined; and on all changes occur after every storm—changes which in some cases result in a gradual advance of the shore line and in others in its recession.

Effects of ice.—In northerly latitudes some peculiar effects of wave action are experienced not encountered in milder climates. One such effect is the formation upon certain classes of engineering structures of huge masses of ice, resulting from the freezing of the water and spray thrown upon the structure by breaking waves.

In many cases this imposes a large additional and eccentric load upon a structure; changes the position of its center of gravity and increases the area of the surface exposed to wave action. Such a case is shown in the photograph on page 223,

which shows a part of the ice remaining on the west breakwater at the upper entrance of Portage Lake canals, Lake Superior, April 19, 1902. The top of the mass shown in the picture was about 12 feet above the deck of the breakwater and about 20 feet above the level of the lake. It had been formed during the preceding winter from water, spray, and blocks of ice dashed up by storm waves. In March, 1902, this breakwater was covered by a solid mass of ice about 2,000 feet long, 30 to 40 feet wide, and 20 to 25 feet above the deck, imposing an extra load of about 20 tons per linear foot upon the pile foundation. By April 19 the mild weather of an unusually early spring and the wash of waves, when the temperature was above freezing, had reduced this covering of ice to the dimensions shown in the photograph. A similar formation of ice takes place here every winter, and so far no damage has resulted, but it can readily be seen from the photograph that at the time it was taken the normal stability of the breakwater against overturning was considerably decreased by the mass of ice upon it, which was situated almost entirely upon the rear half of the deck, and which presented a greatly increased surface to possible wave action. As a rule, a large continuous superimposed mass of firm ice causes no damage to a breakwater or a pier during freezing weather, unless the structure should overturn as a whole (an event but little likely to happen), because, as shown by numerous tests in the laboratory of the United States Engineer Office at Duluth, Minn., clear ice in cold weather has an average tensile strength of about 312 pounds per square inch, and as long as the mass is firm and continuous, its action is similar to that of a monolithic concrete superstructure. When it begins to melt, and is partly washed away by waves, somewhat as shown in the photograph, it frequently becomes a menace to the work.

The formation of ice on the sloping-face concrete superstructure of the breakwater at Marquette, Mich., on the south shore of Lake Superior, is shown on page 224.

The same face of this breakwater is shown on page 214. It will be noticed that the mass of ice on top of the breakwater completely masks the form of the superstructure.

During the break-up of ice in the spring, damage is sometimes caused to the wooden decking of breakwaters and piers

CONCRETE SUPERSTRUCTURE OF BREAKWATER AT MARQUETTE, MICH., COVERED BY ICE IN WINTER.

BLOCKS OF ICE THROWN UP BY WAVES AT MARQUETTE, MICH.

and to other classes of structures by blocks of ice hurled against them by waves.

At the Duluth Canal during a northeast gale March 7, 1902, a block of firm, clear ice, containing about 10 cubic feet, was thrown by waves 20 feet above the level of the lake, breaking off an iron lamp-post at the outer end of the north pier, the point of fracture being 20.25 feet above datum. All of the dynamometers, four in number, at the Duluth Canal and at Superior Entry were struck by blocks of ice thrown against them by the waves, and every dynamometer had repeatedly reached the limit of its spring, as was shown by the fact that the iron recording rods in every case were worn into regular "shoulders" at their limits, which limits for the different dynamometers corresponded to pressures of 4,910, 5,195, 5,225, and 5,360 pounds per square foot, respectively. On this occasion there was no "pack ice," but the surface of the water was covered with floating pieces of firm ice, varying from about 1 to 20 cubic feet in volume.

Just south of the Duluth Canal, parallel to the shore and about 600 yards from it, and at the point where the outer line of breakers met the unbroken ice extending shoreward, an ice ridge was formed during the same storm by blocks of ice cast up by the waves. This ridge extended to the bottom, in water from 15 to 18 feet in depth, and was about 1.25 miles long. Its crest was about 30 feet above the level of the lake. Logs, pieces of wreckage, etc., are also frequently hurled by storm waves against engineering structures, inflicting serious damage, which, however, is generally of a local character, and liable to occur only at long intervals.

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CHAPTER XVI.

Data concerning site. Resistance by dead weight preferable to that by artificial combinations. Stability as affected by specific gravity of masonry. Rough beds and smooth exposed faces desirable in masonry work. Use of concrete. Strength and tightness of molds. Concrete, when strong enough to resist wave action. Use of timber in marine constructions. Force may act from within a structure, and in any direction. "Closing in" unfinished work.

In planning structures which are to be exposed to wave action it is very desirable to secure in advance data upon the following subjects:

(*a*) Frequency and violence of storms.

(*b*) The direction and maximum velocity of storm winds.

(*c*) "Fetch."

(*d*) The direction of wave travel.

(*e*) The maximum height and length of storm waves.

(*f*) Whether or not waves break in advance of, or in the vicinity of, the site during storms.

(*g*) The governing depth in the vicinity.

(*h*) The configuration and character of the bottom.

(*i*) The fluctuation of water level.

(*j*) The movement of ice.

If the structure is a pier (in contradistinction to a breakwater), its direction should be so located, if possible, as to coincide with that of wave travel during prevailing storms. This has a dual advantage, for it not only subjects the structure to less liability to injury by wave action, but also greatly facilitates the entrance of vessels during storms.

If the depth from the shore out increases gradually and uniformly, the direction of wave travel during storms will vary but little, whatever be the direction of storm winds.

Having determined the conditions affecting the site, the next question to be considered is the material to be used in its construction. In many cases upon bodies of fresh water it is optional with the engineer to decide whether he will

resist the action of waves by the dead weight of the material, as in the case of masonry or concrete constructions, or by the strength of materials and fastenings, as in the case of the rock-filled timber cribs so extensively used on the Great Lakes. Were it not for the question of first cost it would be preferable in every case to rely upon the dead weight of the material. Mr. Alan Stevenson has pointed out that in preferring weight to strength we will be aided in passing from observed effects of waves on masses of natural rock to the determination of the mass and form of structure which will be capable of opposing similar forces; but that we are in a different and more difficult position when we attempt to pass from observations of natural phenomena, where weight alone is concerned, to the deduction of the strength of an artificial construction capable of resisting the same forces. Still another reason why mass and weight are preferable to strength of material is that the effect of weight is constant and unchangeable in its nature, while the strength which results even from the best planned and executed structure of combined materials is subject to being impaired by the loosening of fastenings from the incessant action of the waves.

The use of stone and concrete in marine structures is so general that it is proper at this point to call special attention to the effect of the specific gravity of the material upon its stability against displacement by wave action.

Mr. J. H. Darling, United States assistant engineer, has treated this subject very thoroughly in Appendix K K to the Report of the Chief of Engineers, United States Army, for 1901. He shows that against overturning *the volumes of two cubes of different densities but of equal stability are inversely proportional to the cubes of their densities.*

For example, the force required in air to overturn a granite block of cubical form, 5 feet on an edge, and weighing 170 pounds per cubic foot, if acting at the center of one face, is 850 pounds per square foot. For a lighter rock—say, sandstone weighing 140 pounds per cubic foot—to resist overturning from the same unit force the cube would have to be 6.07 feet on an edge and 224 cubic feet in volume instead of 125 feet as in the case of the heavier stone. When immersed in fresh water stone loses about 62 pounds in weight per cubic foot, so that granite of 170 pounds will weigh but 108 pounds,

and the cubical block of 125 cubic feet will be overturned by a force of 540 pounds per square foot. The sandstone previously described will weigh 78 pounds per cubic foot in water, and to resist a force of 540 pounds per square foot would require a cube 6.92 feet on an edge. This block would contain 331 cubic feet, or would be 2.65 times the size of the granite block.

Mr. Darling also shows that *the force required to overturn cubes of the same size but of different materials is directly proportional to the densities of the materials in air, or to their net weights if immersed in water.* For example, two cubes of 125 cubic feet each, or 5 feet on an edge, one of stone weighing 170 pounds per cubic foot and the other 140 pounds, would, if immersed in fresh water, require 540 and 390 pounds per square foot, respectively, to overturn them.

For different sizes of cubes of the same material the force per unit of surface required to overturn them is proportional to the linear dimensions. For example, a cube 4 feet on an edge will withstand without overturning twice as much pressure per unit of surface as one 2 feet on an edge.

All of the foregoing relations and comparisons between cubical blocks of different sizes and densities hold true of similar polyhedrons similarly placed with respect to the force acting upon them, and also approximately to rock broken into riprap, or rubble, when there is a general similarity among the pieces of different sizes.

Blocks deposited in a pile or embankment mutually support and protect one another to some extent, so that each will stand a greater unit force without overturning than would a single isolated block of the same dimensions. The relations previously given for isolated pieces of similar form, but of different densities, may be taken as applying approximately to riprap loosely piled, and less accurately to pieces closely piled.

Resistance to sliding.—Mr. Darling shows that *the volume of two cubes of different densities, but of equal stability against sliding, are inversely proportional to the cubes of their densities.*

It will be noticed that this relation is identical with that already found to apply in the case of overturning. For a given cube, if we compare the force necessary to produce

overturning with that required to produce sliding, we find that when the coefficient of friction is less than unity a cube will slide rather than overturn, but if it is greater than unity the block will be displaced by overturning instead of by sliding. The value of the coefficient of friction depends on the roughness of the block and the character of the support upon which it rests. It may vary from 0.25 to 1.11 or more. If the block is not a cube the question as to whether it will be displaced by overturning or by sliding will depend on the form as well as on the coefficient of friction, the face upon which it rests, and the height of the center of gravity above the base.

Relative sizes of cubes of different densities to give equal resistance against overturning or sliding.

Weight per cubic foot in air.	Relative sizes when in air.	Relative sizes when in water.
Pounds.		
180	1.0	1.0
170	1.2	1.3
160	1.4	1.7
150	1.7	2.4
140	2.1	3.4
130	2.6	5.3

There appears to be but little observed data recorded concerning the relative stability of pieces of stone of different densities when exposed to wave action side by side.

At the lake end of the old south pier of the Duluth Canal granite and sandstone riprap were both lowered to a depth of 11 to 14 feet below the water level by wave action. The riprap of granite was about 27 to 30 cubic feet in size and weighed about 175 pounds per cubic foot. That of sandstone was 80 to 90 cubic feet in size and weighed about 135 pounds per cubic foot.

According to theory the sandstone should have been about four times as large as the granite for equal stability, but in fact was only about three times as large. However, the difference in the shape of the individual pieces of riprap doubtless affected the result too much to permit any very definite conclusion to be drawn from the single instance cited.

Experiment has shown that the coefficient of friction of

masonry for sliding is practically the same in water as in air, and that the smoother the bed the smaller is this coefficient. When the beds were carefully polished it was found that the coefficient of friction was reduced about one-fourth from its value for the same beds roughly dressed. It is therefore obvious that from motives both of economy and efficiency it is inadvisable to expend labor in carefully dressing the beds of masonry to be used in harbor construction.

As shown from dynamometer records at Dunbar and at the Duluth Canal, the wave force parallel to the face of a vertical wall is often greater than the horizontal force exerted by the same waves against the face of the wall. The tendency of the vertical component of the force is to injure or destroy all projections from the face of the wall. Great care should therefore be taken to have the exposed faces of marine structures as true and as smooth as practicable.

Owing to its cheapness, permanency, and to the ease with which it can be molded into blocks of any desired form or size, concrete is peculiarly well adapted for use in marine constructions.

It is, however, rather low in specific gravity as compared with granite or trap rock, seldom exceeding 150 pounds per cubic foot in weight.

In exposed situations it possesses the disadvantage of requiring protection against wave action until it has acquired a certain amount of strength through the process of setting.

In carrying on concrete work in exposed situations the mistake is frequently made of attempting to use molds of insufficient strength. It should be remembered that sudden storms are apt to interrupt the work at any stage, and the molds should therefore be strong enough to withstand ordinarily rough weather and tight enough to prevent the concrete from being washed out before it has had time to set.

It was found by experiments at North Beach, St. Augustine, Fla., that concrete of Portland cement could withstand the action of waves in from two to three days.

The wooden mold of the concrete monolith forming the outer end of groin No. 1, North Beach, was partly destroyed by waves 65 hours after the concrete had been placed, exposing about 92 square feet of unprotected concrete to the action of breaking waves. None of this concrete was washed away,

although a dynamometer, but three or four feet distant, indicated a pressure of 509 pounds per square foot.

At this time, as shown by briquettes made from the same batch of concrete, the tensile strength was about 29 pounds per square inch. At the same place concrete but 48 hours old was slightly washed on its upper surface during this storm.

These results accord fairly well with the investigations made during the construction of the Cherbourg breakwater, which showed that the cohesive strength required in a mortar to withstand the battering action of the waves should be at least 22.75 pounds per square inch.

Owing to its liability to suffer from the ravages of the teredo, timber is used to a very limited extent upon works located in salt water, but on account of its abundance and cheapness heretofore, it has been very extensively used in harbor works on the Great Lakes. Suitable timber for this purpose is now becoming scarce, and its price has risen so much in late years that it appears probable that it will be superseded to an increasing extent hereafter by concrete.

The kind of timber which is most commonly used upon the Great Lakes is white pine. This timber possesses the disadvantage for the use in question of being very light in weight, requiring a greater amount of ballast than would be used with heavier wood. Another disadvantage is that it does not become "water logged," i. e., increase in weight after submergence, even when submerged at considerable depths for many years; for example, the white pine timbers forming the old cribs at the Duluth Canal, when torn apart for removal, floated after thirty years' submergence.

In fresh water, when protected against the chafing action of ice, timber is practically imperishable.

Owing to the buoyancy of timber, the rock-filled timber cribs used upon the Great Lakes have a net weight under water of but about 50 pounds per cubic foot of displacement.

In rock-filled timber crib construction, as used in piers and breakwaters upon the Great Lakes, the integrity of the cribs depends largely upon the decking remaining in place. When the latter is displaced, the stone ballast is washed out of the cribs, the waves act upon both of its walls, and the crib may be destroyed to a depth of 10 or 12 feet below the surface of

the water. Care should therefore be taken to have the decking and its supports of ample strength and securely fastened.

In planning this type of construction, where there are such large interstices in the stone ballast, it should be remembered that it may be exposed to forces acting from within, and that these forces, being often the result of transmitted hydrostatic or dynamic pressures, may act in any direction, tending in some cases to force the decking upward, in others to push out the wall timbers of the crib, and in still others to force off timbers like those forming the sloping face of breakwaters. A concentrated effect of wave action, exerted upon a very limited area, may by hydrostatic pressure develop its force upon an area many times greater.

When timbers are held in place by spikes only, the latter are frequently drawn out by the action of waves. When there is a danger of this occurring, lag screws, or bolts with nuts and washers, should be used, care being taken in particularly exposed places to upset the threads of the screw after the nut is in place.

Injury to marine constructions oftener occurs while work is in progress than after completion. As the more important works of this class occupy more than a single working season during construction, it is very important that when work is interrupted the unfinished end should be carefully protected against the action of storms. Especially is this precaution necessary in the case of structures formed of masonry or concrete blocks, which are liable to be displaced one by one unless properly protected.

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